Problem 26) $\int_0^{\pi} \cos^{2n}\theta\, d\theta = \frac{1}{2}\int_0^{2\pi} \cos^{2n}\theta\, d\theta = \frac{1}{2}\oint_{\text{Unit Circle}} \left(\frac{z + 1/z}{2}\right)^{2n} \frac{dz}{iz}$

$$= \frac{1}{2^{2n+1} i}\oint \frac{(z^2+1)^{2n}}{z^{2n+1}}\, dz = \frac{1}{2^{2n+1} i}\oint_{\text{Unit Circle}} \frac{\sum_{m=0}^{2n} \binom{2n}{m} z^{2m}}{z^{2n+1}}\, dz.$$

Every term in the above sum integrates to zero, except for the term that is proportional to $\frac{1}{z}$. This corresponds to $m=n$. Therefore,

$$\int_0^{\pi} \cos^{2n}\theta\, d\theta = \frac{1}{2^{2n+1} i}\binom{2n}{n}\oint \frac{dz}{z} = \frac{2\pi i}{2^{2n+1} i}\,\frac{(2n)!}{n!\,n!} = \pi\,\frac{(2n)!}{2^{2n}(n!)^2}.$$

Note that $2^n n!$ removes all the even terms from the numerator, $(2n)!$. Thus the numerator becomes $(2n-1)!!$. The remaining $2^n n!$ in the denominator is the product of all even integers up to $2n$, which is $(2n)!!$. Therefore, the final result may be written as $\pi\,\frac{(2n-1)!!}{(2n)!!}$