

Problem 24)

$$\text{Poles: } z^4 + 1 = 0 \Rightarrow z^4 = -1 = e^{i(2n+1)\pi} \Rightarrow z = e^{i(2n+1)\pi/4}$$

There are four different poles as follows:  $z_1 = e^{i\pi/4}$ ,  $z_2 = e^{i3\pi/4}$ ,  $z_3 = e^{i5\pi/4}$

and  $z_4 = e^{i7\pi/4}$ . Only  $z_1$  fall within the chosen contour.

$$\begin{aligned} \text{Residue at } z_1 &= \frac{z_1}{(z_1 - z_2)(z_1 - z_3)(z_1 - z_4)} = \frac{e^{i\pi/4}}{(e^{i\pi/4} - e^{i3\pi/4})(e^{i\pi/4} - e^{i5\pi/4})(e^{i\pi/4} - e^{i7\pi/4})} \\ &= \frac{e^{i\pi/4}}{\sqrt{2} \cdot 2e^{i\pi/4} \cdot \sqrt{2}i} = \frac{1}{4i} \end{aligned}$$

We also note that  $\int \frac{z dz}{z^4 + 1}$  approaches zero as  $R \rightarrow \infty$  on the circular arc.

$$\text{Therefore, } \int_0^{\infty} \frac{x dx}{x^4 + 1} - \int_0^{\infty} \frac{iy}{(iy)^4 + 1} idy = 2\pi i \left( \frac{1}{4i} \right) \Rightarrow$$

$$\int_0^{\infty} \frac{x dx}{x^4 + 1} + \int_0^{\infty} \frac{y dy}{y^4 + 1} = \frac{\pi}{2} \Rightarrow \int_0^{\infty} \frac{x dx}{x^4 + 1} = \frac{\pi}{4}$$