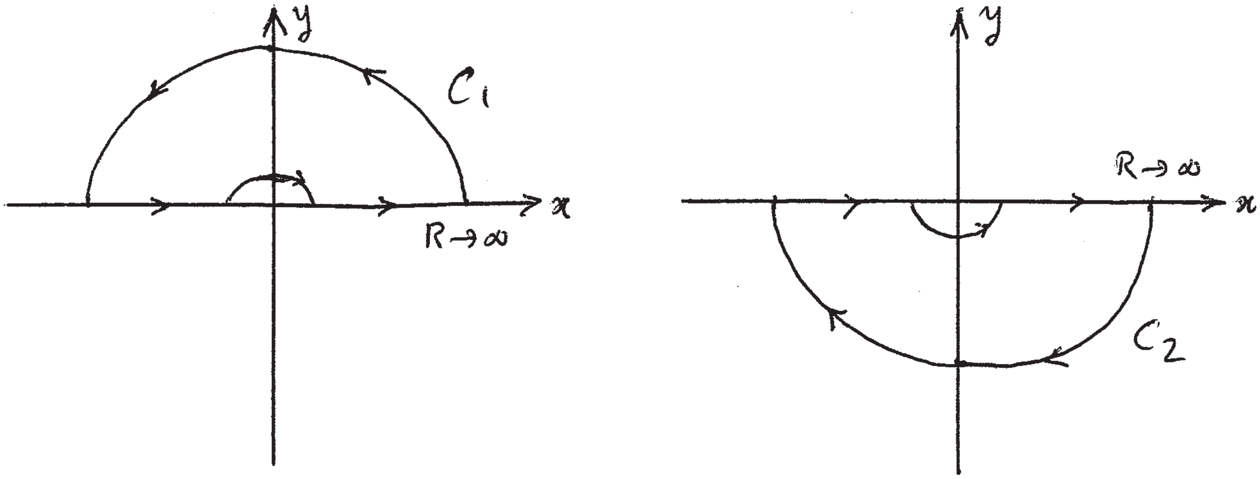
Problem 19) $$\int_{-\infty}^{\infty} \frac{\cos bx - \cos ax}{x^2}\,dx = \frac{1}{2}\int_{-\infty}^{\infty} \frac{e^{ibx} - e^{iax}}{x^2}\,dx + \frac{1}{2}\int_{-\infty}^{\infty}\frac{e^{-ibx} - e^{-iax}}{x^2}\,dx.$$



Since both a and b are positive, the first integral must be evaluated using the contour C_1, while the second integral must be evaluated using C_2. In both cases the integral over the large semi-circle vanishes in the limit $R \to \infty$ (Jordan's lemma). On the small semi-circle we write $z = \epsilon e^{i\theta}$, $dz = i\epsilon e^{i\theta} d\theta$. Since there are no poles inside either contour, we'll have:

$$\int_{-\infty}^{\infty} \frac{e^{ibx} - e^{iax}}{x^2}\,dx = \int_0^{\pi} \frac{e^{ibz} - e^{iaz}}{\epsilon^2 e^{2i\theta}}\, i\epsilon e^{i\theta} d\theta \quad \leftarrow z = \epsilon e^{i\theta}$$

$$= i\int_0^{\pi} \frac{(1 + ibz + \cdots) - (1 + iaz + \cdots)}{\epsilon e^{i\theta}}\,d\theta = i\int_0^{\pi} \frac{i(b-a)\cancel{\epsilon e^{i\theta}} + O(\epsilon^2)}{\cancel{\epsilon e^{i\theta}}}\,d\theta$$

$$\xrightarrow[\epsilon \to 0]{} i^2 (b-a)\pi = (a-b)\pi \Longrightarrow \frac{1}{2}\int_{-\infty}^{\infty} \frac{e^{ibx} - e^{iax}}{x^2}\,dx = \frac{1}{2}\pi(a-b)$$

The second integral is evaluated the same way. The integral over θ goes from π to 2π, but the result must be multiplied with -1, because of the direction of travel around the small semi-circle of C_2. There will be another minus sign because $e^{-ibz} = 1 - ibz + \cdots$ and $e^{-iaz} = 1 - iaz + \cdots$. The two minus signs cancel out. Therefore the two integrals end up being the same.