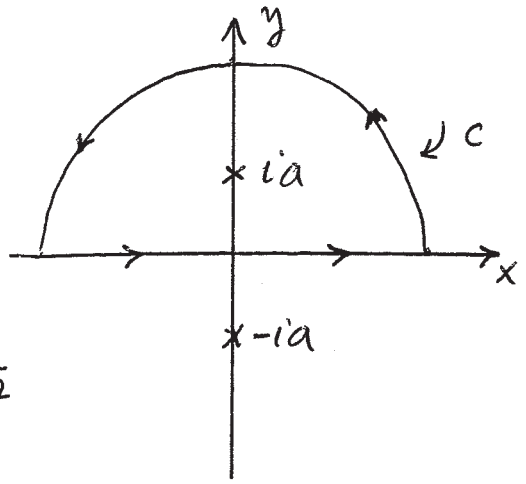
Problem 18)



$$\int_0^\infty \frac{dx}{(x^2+a^2)^2} = \frac{1}{2}\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)^2}$$

$$= \frac{1}{2}\oint_C \frac{dz}{(z^2+a^2)^2} = \frac{1}{2}\oint_C \frac{dz}{(z-ia)^2(z+ia)^2}$$

$$= \frac{1}{2}(2\pi i)\,(\text{Residue at } z_0 = ia).$$

The pole at $z_0 = ia$ is a 2nd-order pole, so the residue is the derivative of $f(z) = \dfrac{1}{(z+ia)^2}$ evaluated at $z = ia$. We have:

$$f'(z)\Big|_{z=ia} = \frac{d}{dz}(z+ia)^{-2}\Big|_{z=ia} = -2(ia+ia)^{-3} = -\frac{2}{(2ia)^3} = \frac{-i}{4a^3}.$$

Therefore, $\displaystyle\int_0^\infty \frac{dx}{(x^2+a^2)^2} = \frac{1}{2}(2\pi i)\left(-\frac{i}{4a^3}\right) = \frac{\pi}{4a^3}.$ ✓