Problem 17) The function $\dfrac{1}{(a+\cos\theta)^2}$ is even, because $\cos\theta = \cos(-\theta)$.

Therefore, the integral from $-\pi$ to zero is equal to that from zero to π. We thus write:

$$\int_0^{\pi} \frac{d\theta}{(a+\cos\theta)^2} = \frac{1}{2}\int_{-\pi}^{\pi} \frac{d\theta}{(a+\cos\theta)^2} = \frac{1}{2}\oint_{\text{Unit Circle}} \frac{\frac{dz}{iz}}{\left(a+\frac{z+z^{-1}}{2}\right)^2}$$

On the unit circle, $z=e^{i\theta}$ and $dz = ie^{i\theta}d\theta$ $\Rightarrow d\theta = dz/(iz)$. Also, $\cos\theta = \frac{z+z^{-1}}{2}$.

$$= -2i\oint_{\text{unit circle}} \frac{z\,dz}{(z^2+2az+1)^2} = -2i\oint_{\text{unit circle}} \frac{z\,dz}{(z-z_1)^2(z-z_2)^2}$$

The roots of the denominator are $z_1, z_2 = -a \pm \sqrt{a^2-1}$. Since $z_1 z_2 = 1$, one of the roots is inside the unit circle, while the other one is outside. Since $a>1$, the root that is inside is $z_1 = -a+\sqrt{a^2-1}$. We thus have:

$$\int_0^{\pi} \frac{d\theta}{(a+\cos\theta)^2} = -2i\oint_{\text{Unit Circle}} \frac{f(z)}{(z-z_1)^2}dz \quad \text{where } f(z) = \frac{z}{(z-z_2)^2}.$$

Since z_1 is a 2^{nd}-order pole, the residue is $f'(z)\big|_{z=z_1} = \frac{d}{dz}\left[z(z-z_2)^{-2}\right]_{z=z_1}$

$$= (z_1-z_2)^{-2} - 2z_1(z_1-z_2)^{-3} = -(z_1-z_2)^{-3}(z_1+z_2) = 2a\left(2\sqrt{a^2-1}\right)^{-3}$$

$$= \frac{a}{4(a^2-1)^{3/2}}.$$ Multiplication with $2\pi i$ then yields the integral around the unit circle. We thus have:

$$\int_0^{\pi} \frac{d\theta}{(a+\cos\theta)^2} = -2i(2\pi i)\frac{a}{4(a^2-1)^{3/2}} = \frac{\pi a}{(a^2-1)^{3/2}}.$$