Problem 16) $f(z) = (z-z_0)\,g(z) \Rightarrow \dfrac{df(z)}{dz} = g(z) + (z-z_0)\dfrac{dg(z)}{dz} \Rightarrow$

$$\left.\frac{df(z)}{dz}\right|_{z=z_0} = g(z_0)\,. \quad \text{Therefore} \quad \left.\frac{z-z_0}{f(z)}\right|_{z=z_0} = \left.\frac{1}{g(z)}\right|_{z=z_0} = \frac{1}{g(z_0)} = \frac{1}{\left.\frac{df(z)}{dz}\right|_{z=z_0}}\,.$$

Let $f(z) = z^3+1$. Then the above procedure yields:

$$\left.\frac{z-z_0}{f(z)}\right|_{z_0=-1} = \frac{1}{\left.3z^2\right|_{z=-1}} = \frac{1}{3}\,.$$

Alternatively, $z^3+1=0 \Rightarrow z^3 = -1 = e^{i\pi+2in\pi} \Rightarrow z = e^{i\pi(2n+1)/3}$

$\Rightarrow z_1 = e^{i\pi/3}$, $z_0 = e^{i\pi} = -1$, $z_2 = e^{-i\pi/3} \leftarrow (n=-1)$.

$\nearrow (n=0)$ $\quad\quad \nearrow (n=1)$

Therefore, $z^3+1 = (z-z_0)(z-z_1)(z-z_2) = (z+1)(z-e^{i\pi/3})(z-e^{-i\pi/3})$.

We have: $$\left.\frac{z-z_0}{f(z)}\right|_{z=z_0} = \left.\frac{\cancel{z+1}}{(\cancel{z+1})(z-e^{i\pi/3})(z-e^{-i\pi/3})}\right|_{z=-1}$$

$$= \frac{1}{(1+e^{i\pi/3})(1+e^{-i\pi/3})} = \frac{1}{1+1+e^{i\pi/3}+e^{-i\pi/3}} = \frac{1}{2+2\cos(\pi/3)}$$

$$= \frac{1}{2+2\left(\frac{1}{2}\right)} = \frac{1}{3}. \checkmark$$

The direct calculation thus yields the same result as that obtained with the indirect method using the derivative of $f(z)$ at $z=z_0$. The indirect method, of course, is much simpler.