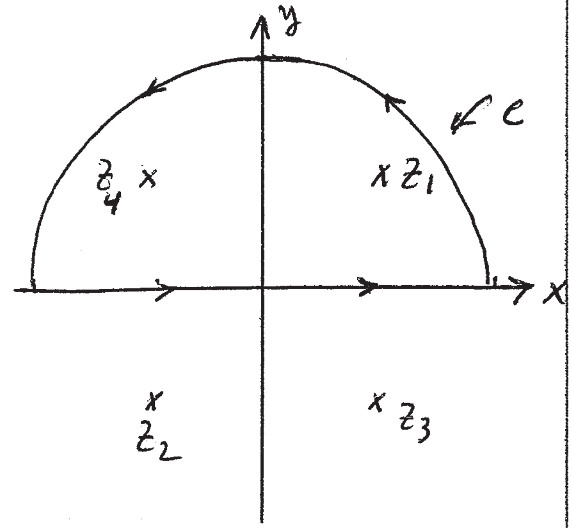
Problem 15)



$$\int_{-\infty}^{\infty} \frac{x^2}{1+x^4}\,dx = \oint_C \frac{z^2}{1+z^4}\,dz$$

$$= \oint_C \frac{z^2}{(z-z_1)(z-z_2)(z-z_3)(z-z_4)}.$$

The poles are obtained by solving

$$z^4+1=0 \Rightarrow z^2 = \pm i = e^{\pm i\pi/2}$$

$$\Rightarrow z = \pm e^{\pm i\pi/4} \Rightarrow z_1 = e^{i\pi/4},\ z_2 = -e^{i\pi/4},\ z_3 = e^{-i\pi/4},\ z_4 = -e^{-i\pi/4}.$$

The only poles inside the contour C are z_1 and z_4. These are first-order poles, so we have:

$$\int_{-\infty}^{\infty} \frac{x^2}{1+x^4}\,dx = 2\pi i\left(\text{Residue at } z_1 + \text{Residue at } z_4\right)$$

$$= \frac{2\pi i\, z_1^2}{(z_1-z_2)(z_1-z_3)(z_1-z_4)} + \frac{2\pi i\, z_4^2}{(z_4-z_1)(z_4-z_2)(z_4-z_3)}$$

$$= \frac{2\pi i\, e^{i\pi/2}}{(e^{i\pi/4}+e^{i\pi/4})(e^{i\pi/4}-e^{-i\pi/4})(e^{i\pi/4}+e^{-i\pi/4})}$$

$$+ \frac{2\pi i\, e^{-i\pi/2}}{(-e^{-i\pi/4}-e^{i\pi/4})(-e^{-i\pi/4}+e^{i\pi/4})(-e^{-i\pi/4}-e^{-i\pi/4})}$$

$$= \frac{2\pi i\, e^{i\pi/2}}{(2e^{i\pi/4})(2i\sin\pi/4)(2\cos\pi/4)} + \frac{2\pi i\, e^{-i\pi/2}}{(-2\cos\pi/4)(2i\sin\pi/4)(-2e^{-i\pi/4})}$$

$$= \frac{\pi e^{+i\pi/4}}{4\sin\frac{\pi}{4}\cos\frac{\pi}{4}} + \frac{\pi e^{-i\pi/4}}{4\sin\frac{\pi}{4}\cos\frac{\pi}{4}} = \frac{2\pi\cos\pi/4}{4\sin\frac{\pi}{4}\cos\frac{\pi}{4}} = \frac{\pi}{2\sin\frac{\pi}{4}} = \frac{\pi}{\sqrt{2}}.$$