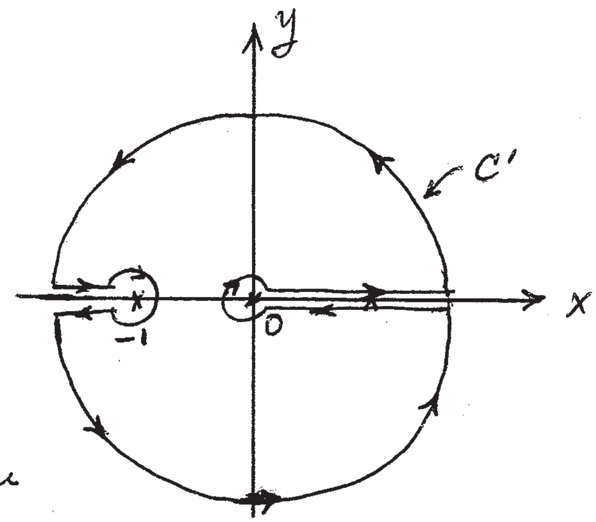
Problem 9)



$$\oint_C \frac{dz}{z^2+z} = \oint_C \frac{dz}{z(z+1)}$$

The integral over the contour C' shown in the figure is zero, because the function $\frac{1}{z(z+1)}$ is analytic everywhere inside and on this contour. The integrals over the straight-line segments of the contour cancel each other out. The two small circles are traversed clockwise. Thus the integral over the circle of radius $R>1$ is equal to the sum of the integrals over the two small circles, provided that these small circles are traversed counterclockwise. For the circle surrounding $z=0$ we write $z=\epsilon e^{i\theta}$.

Then $$\oint_{\substack{\text{circle}\\ \text{at } z=0}} \frac{i\cancel{\epsilon e^{i\theta}}\,d\theta}{\cancel{\epsilon e^{i\theta}}(\epsilon e^{i\theta}+1)} \xrightarrow[\epsilon\to 0]{} \oint_0^{2\pi} i\,d\theta = 2\pi i$$

For the circle surrounding $z=-1$ we write $z=-1+\epsilon e^{i\theta}$. We then have $$\oint_{\substack{\text{circle}\\ \text{at } z=-1}} \frac{dz}{z(z+1)} = \oint_0^{2\pi} \frac{i\cancel{\epsilon e^{i\theta}}\,d\theta}{(-1+\epsilon e^{i\theta})\cancel{\epsilon e^{i\theta}}} \xrightarrow[\epsilon\to 0]{} -2\pi i$$

The sum of the two integrals is thus equal to zero. ✓

Alternatively, we could write $\oint_C \frac{dz}{z(z+1)} = \oint_C \left(\frac{1}{z} - \frac{1}{z+1}\right)dz = \oint_C \frac{dz}{z} - \oint_C \frac{dz}{z+1}$.

Each integral is then equal to $2\pi i$ times the residue at the corresponding pole, which is equal to 1. The final result is $2\pi i - 2\pi i = 0$, as before.