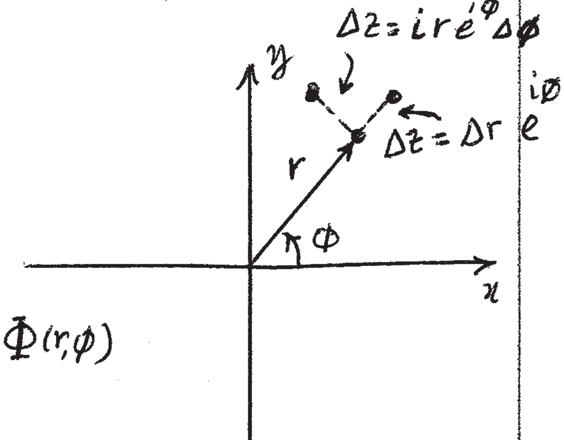
Problem 6)



$$f'(z) = \frac{f(z+\Delta z) - f(z)}{\underset{\to 0}{\Delta z}}$$

a) Let $\Delta z = \Delta r\, e^{i\phi}$. Since $f(z) = R(r,\phi)e^{i\Phi(r,\phi)}$
we may write:

$$f'(z) = \frac{R(r+\Delta r,\phi)\, e^{i\Phi(r+\Delta r,\phi)} - R(r,\phi)e^{i\Phi(r,\phi)}}{\underset{\to 0}{\Delta r}\, e^{i\phi}}$$

$$= e^{-i\phi}\frac{\partial}{\partial r}\left[R(r,\phi)e^{i\Phi(r,\phi)}\right] = e^{-i\phi}\left[\left(\frac{\partial R}{\partial r}\right)e^{i\Phi} + iR\left(\frac{\partial\Phi}{\partial r}\right)e^{i\Phi}\right]$$

b) Next we let $\Delta z = ire^{i\phi}\Delta\phi$, which is a small change in z in the ϕ direction. The derivative of $f(z)$ may now be written:

$$f'(z) = \frac{R(r,\phi+\Delta\phi)e^{i\Phi(r,\phi+\Delta\phi)} - R(r,\phi)e^{i\Phi(r,\phi)}}{ire^{i\phi}\underset{\to 0}{\Delta\phi}}$$

$$= \frac{-ie^{-i\phi}}{r}\frac{\partial}{\partial\phi}\left[R(r,\phi)e^{i\Phi(r,\phi)}\right] = -\frac{ie^{-i\phi}}{r}\left[\left(\frac{\partial R}{\partial\phi}\right)e^{i\Phi} + iR\left(\frac{\partial\Phi}{\partial\phi}\right)e^{i\Phi}\right]$$

Now, the two answers found in (a) and (b) above for $f'(z)$ must be the same. Since $e^{-i\phi}e^{i\Phi}$ multiplies both expressions, all we need to do is set the remaining parts equal to each other, i.e.,

$$\left(\frac{\partial R}{\partial r}\right) + iR\left(\frac{\partial\Phi}{\partial r}\right) = -\frac{i}{r}\left(\frac{\partial R}{\partial\phi}\right) + \frac{R}{r}\left(\frac{\partial\Phi}{\partial\phi}\right).$$ Consequently

$$\frac{\partial R}{\partial r} = \frac{R}{r}\frac{\partial\Phi}{\partial\phi} \quad \text{and} \quad R\frac{\partial\Phi}{\partial r} = -\frac{1}{r}\frac{\partial R}{\partial\phi}.$$