Problem 3) $f'(z) = lim_{\Delta z \to 0}\,[f(z+\Delta z) - f(z)]/\Delta z$

$$= lim_{\Delta z \to 0}\,[(z+\Delta z)^n - z^n]/\Delta z$$

binomial expansion →

$$= lim_{\Delta z \to 0}\left[\sum_{k=0}^{n} \binom{n}{k} z^{n-k}(\Delta z)^k - z^n\right]\Big/\Delta z$$

$$= lim_{\Delta z \to 0}\left[\binom{n}{1} z^{n-1}\Delta z + \sum_{k=2}^{n} \binom{n}{k} z^{n-k}(\Delta z)^k\right]\Big/\Delta z$$

$$= lim_{\Delta z \to 0}\left[n z^{n-1} + \sum_{k=2}^{n} \binom{n}{k} z^{n-k}(\Delta z)^{k-1}\right]$$

← $(\Delta z)^{k-1} \to 0$ for $k \geq 2$

$$= n z^{n-1}.$$

Since the derivative of $f(z) = z^n$ is well-defined as $f'(z) = nz^{n-1}$ at all values z, the function $f(z)$ is analytic throughout the entire z-plane.