Problem 7) For a given value of n, we are asked to maximize the product $x_1 x_2 \cdots x_n$ subject to the constraint $x_1 + x_2 + \cdots + x_n = L$. In accordance with the method of Lagrange multipliers, we form the function $f(x_1, x_2, \cdots, x_n) = x_1 x_2 \cdots x_n + \lambda(x_1 + x_2 + \cdots + x_n)$, then set its derivatives with respect to $x_1, x_2, \cdots, x_n$ equal to zero, as follows:

$$\partial f / \partial x_1 = x_2 x_3 \cdots x_n + \lambda = 0 \quad \rightarrow \quad x_2 x_3 \cdots x_n = -\lambda;$$

$$\partial f / \partial x_2 = x_1 x_3 \cdots x_n + \lambda = 0 \quad \rightarrow \quad x_1 x_3 \cdots x_n = -\lambda;$$

$$\vdots$$

$$\partial f / \partial x_n = x_1 x_2 \cdots x_{n-1} + \lambda = 0 \quad \rightarrow \quad x_1 x_2 \cdots x_{n-1} = -\lambda.$$

Dividing the first of the above equations by the second yields $x_2 / x_1 = 1$, revealing that x_1 and x_2 must be equal. Similarly, dividing the first equation by the third shows that $x_1 = x_3$, and so on. The solution of the above equations is thus given by $x_1 = x_2 = \cdots = x_n = (-\lambda)^{1/(n-1)}$. In the next step, we substitute the above solution into the constraint equation $x_1 + x_2 + \cdots + x_n = L$ to determine the value of λ. We will have $n(-\lambda)^{1/(n-1)} = L$, which yields $\lambda = -(L/n)^{n-1}$. The optimum solution for the lengths of the various segments is thus found to be $x_1 = x_2 = \cdots = x_n = (-\lambda)^{1/(n-1)} = L/n$.