
Problem 21)

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots \quad (-1 < x \leq 1)$$

$$\ln(1-x) = -(x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \dots).$$

$$\rightarrow \ln(1+x) - \ln(1-x) = 2(x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \dots)$$

$$\rightarrow \ln\left(\frac{1+x}{1-x}\right) = 2(x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \dots).$$

$$\frac{1+x}{1-x} = y \rightarrow x = \frac{y-1}{y+1} \rightarrow \ln y = 2\left[\left(\frac{y-1}{y+1}\right) + \frac{1}{3}\left(\frac{y-1}{y+1}\right)^3 + \frac{1}{5}\left(\frac{y-1}{y+1}\right)^5 + \dots\right]; \quad (y > 0).$$
