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**Problem 20)**

$$\begin{aligned}\int_0^{\infty} \ln(1 + e^{-x}) dx &= \int_0^{\infty} \left[ \sum_{n=1}^{\infty} (-1)^{n+1} e^{-nx} / n \right] dx = \sum_{n=1}^{\infty} (-1)^n n^{-2} e^{-nx} \Big|_{x=0}^{\infty} \\ &= \sum_{n=1}^{\infty} (-1)^{n+1} / n^2 = \pi^2 / 12. \quad (\text{G\&R 4.223-1})\end{aligned}$$

$$\begin{aligned}\int_0^{\infty} \ln(1 - e^{-x}) dx &= - \int_0^{\infty} \sum_{n=1}^{\infty} (e^{-nx} / n) dx = \sum_{n=1}^{\infty} n^{-2} e^{-nx} \Big|_{x=0}^{\infty} \\ &= - \sum_{n=1}^{\infty} 1/n^2 = -\pi^2/6. \quad (\text{G\&R 4.223-2})\end{aligned}$$

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