Problem 20)

$$\int_0^\infty \ln(1+e^{-x})\mathrm{d}x = \int_0^\infty \left[\sum_{n=1}^\infty (-1)^{n+1} e^{-nx}/n\right]\mathrm{d}x = \sum_{n=1}^\infty (-1)^n n^{-2} e^{-nx}\Big|_{x=0}^\infty$$

$$= \sum_{n=1}^\infty (-1)^{n+1}/n^2 = \pi^2/12.$$ (G&R **4.223**-1)

$$\int_0^\infty \ln(1-e^{-x})\mathrm{d}x = -\int_0^\infty \sum_{n=1}^\infty (e^{-nx}/n)\,\mathrm{d}x = \sum_{n=1}^\infty n^{-2} e^{-nx}\Big|_{x=0}^\infty$$

$$= -\sum_{n=1}^\infty 1/n^2 = -\pi^2/6.$$ (G&R **4.223**-2)