Problem 14) Using the binomial expansion $(x+y)^n = \sum_{m=0}^{n} \binom{n}{m} x^{n-m} y^m$, we write

$$2^n = (1+1)^n = \sum_{m=0}^{n} \binom{n}{m} = \sum_{m=0,2,4,6,\cdots}^{n} \binom{n}{m} + \sum_{m=1,3,5,7,\cdots}^{n} \binom{n}{m}. \qquad (1)$$

$$0 = (1-1)^n = \sum_{m=0}^{n} \binom{n}{m} (-1)^m = \sum_{m=0,2,4,6,\cdots}^{n} \binom{n}{m} - \sum_{m=1,3,5,7,\cdots}^{n} \binom{n}{m}. \qquad (2)$$

Adding Eqs.(1) and (2) yields the sum over even values of m as $½(2^n + 0) = 2^{n-1}$. Similarly, subtracting Eq.(2) from Eq.(1) yields the sum over odd values of m as $½(2^n - 0) = 2^{n-1}$.