Problem 10)

a) $\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$

b) $\exp(\mathrm{i}x) = 1 + \mathrm{i}x + \frac{(\mathrm{i}x)^2}{2!} + \frac{(\mathrm{i}x)^3}{3!} + \cdots + \frac{(\mathrm{i}x)^n}{n!} + \cdots$

$$= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right) + \mathrm{i}\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right)$$

Considering that $\exp(\mathrm{i}x) = \cos x + \mathrm{i}\sin x$, it is clear that, in the preceding expression, the first sum (the real part) is equal to $\cos x$, while the second sum (the imaginary part) is equal to $\sin x$. Setting $x = 1$ radian, we will have

c) $\cos(1) = 0.54030231 = 1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \frac{1}{8!} - \cdots$

d) $\sin(1) = 0.84147098 = 1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \frac{1}{9!} - \cdots$