Problem 9)

Binomial expansion: $(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k.$

Integrating the above equation over x from 0 to 1 yields

$$\int_0^1 (1+x)^n \mathrm{d}x = \sum_{k=0}^{n} \binom{n}{k} \int_0^1 x^k \mathrm{d}x \quad \rightarrow \quad \left. \frac{(1+x)^{n+1}}{n+1} \right|_{x=0}^{1} = \sum_{k=0}^{n} \binom{n}{k} \left. \frac{x^{k+1}}{k+1} \right|_{x=0}^{1}$$

$$\rightarrow \quad \frac{2^{n+1}-1}{n+1} = \sum_{k=0}^{n} \frac{1}{k+1} \binom{n}{k}.$$