Problem 8) $f(x) = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$; $|x|<1$ ← Geometric Series

Taking derivatives from both sides of the above equation yields:

$$f'(x) = \sum_{n=0}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2} \Rightarrow \sum_{n=1}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2}$$

← because the first term at $n=0$ is zero.

Multiplying both sides by x then yields:

$$\sum_{n=1}^{\infty} n x^n = \frac{x}{(1-x)^2} \quad ; \quad |x|<1$$