

Problem 6)

Since all the terms of zeta-function are positive, we can rearrange them. Separating the odd terms from the even terms, we find:

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \sum_{n=1}^{\infty} \frac{1}{(2n)^4} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \frac{1}{16} \sum_{n=1}^{\infty} \frac{1}{n^4} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} \Rightarrow$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \left(1 - \frac{1}{16}\right) \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{15}{16} \times \frac{\pi^4}{90} = \frac{\pi^4}{96} \quad \checkmark$$