Problem 4)

$$\frac{\sin x\cos x}{x}=\prod_{n=1}^{\infty}\left(1-\frac{x^2}{n^2\pi^2}\right)\prod_{n=1}^{\infty}\left(1-\frac{4x^2}{(2n-1)^2\pi^2}\right)=\prod_{n=1}^{\infty}\left(1-\frac{x^2}{n^2\pi^2}\right)\left(1-\frac{4x^2}{(2n-1)^2\pi^2}\right)$$

$$=\prod_{n=1}^{\infty}\left(1-\frac{4x^2}{(2n)^2\pi^2}\right)\left(1-\frac{4x^2}{(2n-1)^2\pi^2}\right)=\prod_{n=1}^{\infty}\left(1-\left(\frac{2x}{n\pi}\right)^2\right)=\frac{\sin(2x)}{2x}.$$

Here we have taken advantage of the fact that, as n moves through the positive integers from 1 to ∞, the odd and even integers $2n-1$ and $2n$ also cover the entire range from 1 to ∞. The last infinite product has the same form as the expansion of $\sin x/x$, except for the appearance of $2x$ in place of x. That is why we have set it equal to $\sin(2x)/(2x)$.