Problem 3)

$$\exp(x)\exp(y) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \sum_{m=0}^{\infty} \frac{y^m}{m!} = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{x^n y^m}{n!m!}$$

$$= \sum_{k=0}^{\infty} \sum_{n=0}^{k} \frac{x^n y^{k-n}}{n!(k-n)!} = \sum_{k=0}^{\infty} \frac{1}{k!} \sum_{n=0}^{k} \binom{k}{n} x^n y^{k-n} \qquad \leftarrow \boxed{k = n + m}$$

$$= \sum_{k=0}^{\infty} \frac{(x+y)^k}{k!} = \exp(x+y).$$