

Problem 2) a) 
$$\sum_{m=2}^{\infty} [f^{(m)} - 1] = \sum_{m=2}^{\infty} \left( \sum_{n=1}^{\infty} \frac{1}{n^m} - 1 \right) = \sum_{m=2}^{\infty} \sum_{n=2}^{\infty} \frac{1}{n^m}$$

Changing the order of summation over  $n$  and  $m$ , we'll have

$$\begin{aligned} \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{1}{n^m} &= \sum_{n=2}^{\infty} \frac{1/n^2}{1 - 1/n} = \sum_{n=2}^{\infty} \frac{1}{n^2 - n} = \sum_{n=2}^{\infty} \frac{-1}{n} + \frac{1}{n-1} \\ &= -\sum_{n=2}^{\infty} \frac{1}{n} + \sum_{n=1}^{\infty} \frac{1}{n} = -\cancel{\sum_{n=2}^{\infty} \frac{1}{n}} + \left( 1 + \cancel{\sum_{n=2}^{\infty} \frac{1}{n}} \right) = 1 \quad \checkmark \end{aligned}$$

b) 
$$\sum_{m=2}^{\infty} (-1)^m [f^{(m)} - 1] = \sum_{m=2}^{\infty} (-1)^m \sum_{n=2}^{\infty} \frac{1}{n^m} = \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{(-1)^m}{n^m} \leftarrow \text{geometric series}$$

$$\begin{aligned} &= \sum_{n=2}^{\infty} \frac{1/n^2}{1 + 1/n} = \sum_{n=2}^{\infty} \frac{1}{n^2 + n} = \sum_{n=2}^{\infty} \frac{1}{n} - \frac{1}{n+1} = \sum_{n=2}^{\infty} \frac{1}{n} - \sum_{n=3}^{\infty} \frac{1}{n} = \frac{1}{2} \quad \checkmark \end{aligned}$$