Problem 10)

$$
\begin{aligned}
z_1^{z_2} &= [\exp(\ln z_1)]^{z_2} \\
&= \exp(z_2 \ln z_1) \\
&= \exp\left[z_2 \ln\left(|z_1| e^{i\varphi_1}\right)\right] \\
&= \exp\left\{z_2 \ln\left[|z_1| e^{i(\varphi_1 + 2n\pi)}\right]\right\} \quad \leftarrow n \text{ an arbitrary integer} \\
&= \exp\{z_2[\ln|z_1| + i(\varphi_1 + 2n\pi)]\} \\
&= \exp(z_2 \ln|z_1|) \times \exp(iz_2\varphi_1) \times \exp(i2n\pi z_2)
\end{aligned}
$$

In general, there exist an infinite number of values for $z_1^{z_2}$, each corresponding to a different value of the integer n. However, if z_2 happens to be an integer, the last exponential factor, $\exp(i2n\pi z_2)$, will be equal to 1.0 for all values of n, in which case $z_1^{z_2}$ will be uniquely specified. If z_2 happens to be an irreducible rational m_1/m_2, there will be m_2 distinct values of $z_1^{z_2}$, corresponding to $n = 0, 1, 2, \dots, (m_2 - 1)$.

Defining a function $f(z) = z^{z_2}$ for a non-integer z_2 requires the identification of a branch-cut, so that the function has a unique value for each z. Similar considerations apply to the function $g(z) = z^z$.