

Problem 9)

a) $f(z) = \tan^{-1}(z) \rightarrow \tan f(z) = z \rightarrow \frac{\sin f(z)}{\cos f(z)} = z \rightarrow \frac{(e^{if} - e^{-if})/2i}{(e^{if} + e^{-if})/2} = z$

$\rightarrow \frac{e^{i2f} - 1}{e^{i2f} + 1} = iz \rightarrow e^{i2f} - 1 = iz e^{i2f} + iz \rightarrow e^{i2f}(1 - iz) = 1 + iz$

$\rightarrow e^{i2f(z)} = \frac{1 + iz}{1 - iz} \rightarrow i2f(z) = \ln\left(\frac{1 + iz}{1 - iz}\right) = \ln\left(\frac{i - z}{i + z}\right) \rightarrow f(z) = -\left(\frac{i}{2}\right) \ln\left(\frac{i - z}{i + z}\right)$

$\rightarrow f(z) = \left(\frac{i}{2}\right) \ln\left(\frac{i + z}{i - z}\right).$

b) $f(z) = \tanh^{-1}(z) \rightarrow \tanh f(z) = z \rightarrow \frac{\sinh f(z)}{\cosh f(z)} = z \rightarrow \frac{(e^f - e^{-f})/2}{(e^f + e^{-f})/2} = z$

$\rightarrow \frac{e^{2f} - 1}{e^{2f} + 1} = z \rightarrow e^{2f} - 1 = z e^{2f} + z \rightarrow e^{2f(z)}(1 - z) = 1 + z$

$\rightarrow e^{2f(z)} = \frac{1 + z}{1 - z} \rightarrow f(z) = \frac{1}{2} \ln\left(\frac{1 + z}{1 - z}\right).$
