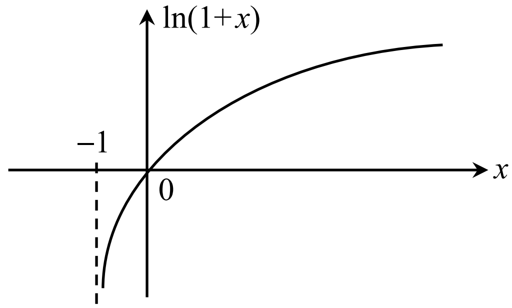
Problem 4)

a)



$$\left.\frac{d\ln(1+x)}{dx}\right|_{x=0} = (1+x)^{-1}|_{x=0} = 1$$

$$\left.\frac{d^2\ln(1+x)}{dx^2}\right|_{x=0} = -(1+x)^{-2}|_{x=0} = -1$$

$$\left.\frac{d^3\ln(1+x)}{dx^3}\right|_{x=0} = 2(1+x)^{-3}|_{x=0} = 2!$$

$$\left.\frac{d^4\ln(1+x)}{dx^4}\right|_{x=0} = -3!\,(1+x)^{-4}|_{x=0} = -3!$$

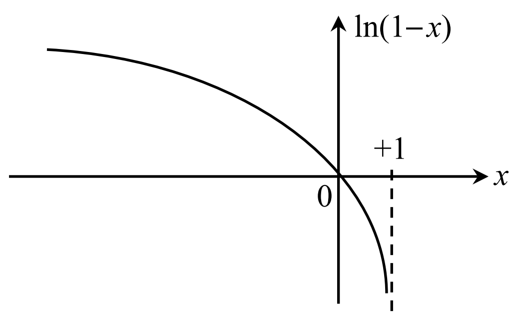
$$\vdots$$

Therefore,

$$\ln(1+x) = \ln(1) + x - \frac{1\times x^2}{2!} + \frac{2!\times x^3}{3!} - \frac{3!\times x^4}{4!} + \cdots$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots.$$

b)

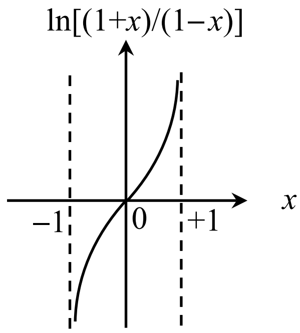


In this case the solution is obtained by changing the sign of x in the Taylor-series expansion of $\ln(1+x)$ obtained in part (a), that is,

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots.$$

c) $\ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x) = \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots\right) - \left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots\right)$

$$= 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \cdots\right).$$



d) Setting $x = ½$ in the above Taylor-series expansion of $\ln\left(\frac{1+x}{1-x}\right)$, we find

$$\ln\left(\frac{1+x}{1-x}\right)\bigg|_{x=½} = \ln\left(\frac{3/2}{1/2}\right) = \ln 3 = 2\left(\frac{1}{2} + \frac{1}{3\times 2^3} + \frac{1}{5\times 2^5} + \frac{1}{7\times 2^7} + \cdots\right).$$

e) Setting $x = ⅓$ in the Taylor-series expansion of $\ln\left(\frac{1+x}{1-x}\right)$, we find

$$\ln\left(\frac{1+x}{1-x}\right)\bigg|_{x=⅓} = \ln\left(\frac{4/3}{2/3}\right) = \ln 2 = 2\left(\frac{1}{3} + \frac{1}{3\times 3^3} + \frac{1}{5\times 3^5} + \frac{1}{7\times 3^7} + \cdots\right).$$