Problem 20) Dropping the perpendicular line DE from D onto AB, as in figure (a), we will have

$$\tan\theta = \overline{ED}/\overline{EB} = \overline{ED}/(\overline{AB} - \overline{AE}) = \overline{AD}\sin 20°/(\overline{AB} - \overline{AD}\cos 20°). \qquad (1)$$

Dropping the perpendicular bisector AF from A onto BC, as in figure (b), we will have

$$\sin 10° = \overline{BF}/\overline{AB} = \overline{BC}/(2\overline{AB}) = \overline{AD}/(2\overline{AB}) \quad \rightarrow \quad \overline{AD} = 2\overline{AB}\sin 10°. \qquad (2)$$

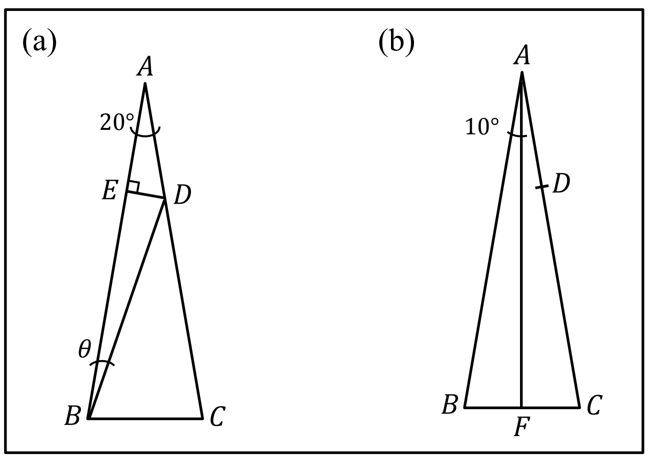
Substitution into Eq.(1) now yields

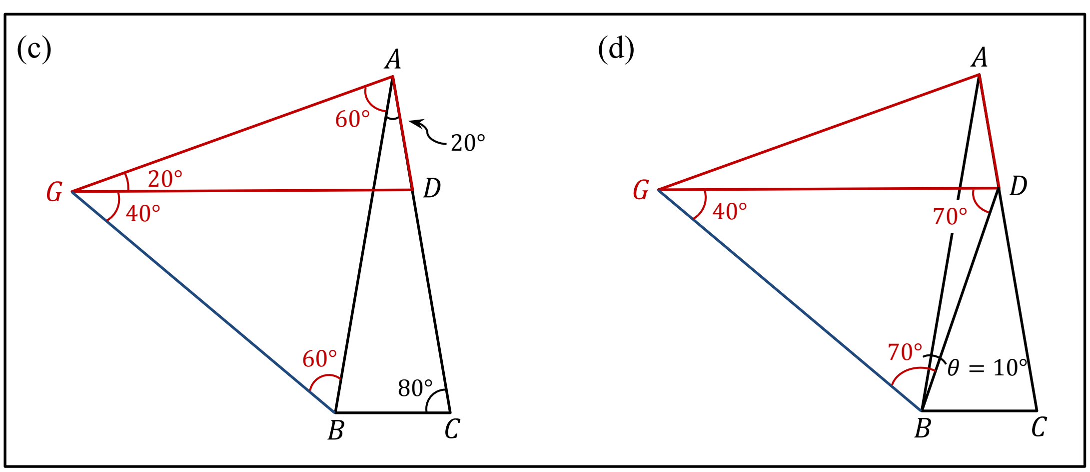
$$\tan\theta = \frac{2\sin 10°\sin 20°}{1-2\sin 10°\cos 20°}. \qquad (3)$$

The formula is completely general and applies to any isosceles triangle having angle α at its vertex A, namely,

$$\tan\theta = \frac{2\sin(\alpha/2)\sin\alpha}{1-2\sin(\alpha/2)\cos\alpha}. \qquad (4)$$

This is true irrespective of whether $\alpha \le 60°$, in which case D lies between A and C, or $60° < \alpha \le 180°$, in which case D lies on the extension of AC beyond the vertex C. In the special case of $\alpha = 20°$, the solution is simplified by writing 1 in the denominator as follows:





$$1 = 2\sin 30° = 2\sin(10° + 20°) = 2\sin 10° \cos 20° + 2\cos 10° \sin 20°. \qquad (5)$$

Substitution into Eq.(3) now yields $\tan\theta = \tan 10°$, and, therefore, $\theta = 10°$. This result also emerges from a fully geometrical treatment of the problem, as follows. With reference to figure (c), construct the GDA triangle over the base AD to be identical with the ABC triangle. The angle GAD is 80° and, therefore, the angle GAB is 60°. Considering that $\overline{AG} = \overline{AB}$, the isosceles triangle AGB with an apex angle of 60° must be equilateral. Consequently, $\overline{GB} = \overline{GA}$ and, therefore, $\overline{GB} = \overline{GD}$. Now, referencing figure (d), since the isosceles triangle GBD has an apex angle of 40°, the angle GBD must be 70°, which reveals that the angle θ is 10°.