Problem 13) The original triangle has side-length L and area $\sqrt{3}L^2/4$. The first triangle that is removed has side-length $L/2$ and area $\sqrt{3}L^2/16$. The next three triangles that are removed have side-lengths $L/4$ and a total area of $3\sqrt{3}L^2/64$. Thus, in each step, the number of removed triangles increases by a factor of 3, while the area of each triangle decreases by a factor of 4. We will have

$$\text{Area removed} = \left(\sqrt{3}L^2/16\right)[1 + (¾) + (¾)^2 + (¾)^3 + \cdots] = \frac{\sqrt{3}L^2/16}{1-(3/4)} = \sqrt{3}L^2/4.$$

It is seen that the total area removed is equal to the area of the original triangle. As for the total length of the boundary after n steps of the removal process, we will have

$$\begin{aligned}\text{Length of boundary} &= 3L + (3L/2) + (9L/4) + (27L/8) + \cdots + (3^nL/2^n)\\ &= 3L + (3L/2)[1 + (3/2) + (3/2)^2 + \cdots + (3/2)^{n-1}]\\ &= 3L + (3L/2)\left[\frac{(3/2)^n - 1}{(3/2) - 1}\right] = 3(3/2)^nL.\end{aligned}$$

The length of the boundary is seen to increase exponentially with the number n of removal steps.