

Problem 8) a) Let AA' be the altitude dropped from the vertex A onto the opposite side BC . We will have $\overline{AA'} = \overline{AB} \sin B = \overline{AC} \sin C$. The area of the triangle is one-half the length of the base BC times the altitude AA' . Therefore,

$$\begin{aligned} \text{Area of } ABC &= \frac{1}{2} \overline{BC} \times \overline{AA'} = \frac{1}{2} \overline{BC} \times \overline{AB} \sin B = \frac{1}{2} \overline{BC} \times \overline{AC} \sin C \\ &= \frac{1}{2} ac \sin B = \frac{1}{2} ab \sin C. \end{aligned}$$

The remaining equality can be proven by dropping the altitude BB' from the vertex B onto the opposite side AC , then repeating the above argument.

b) Upon dividing the identity obtained in part (a) by $\frac{1}{2}abc$, we will arrive at the desired relation:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$
