

Problem 7)

$$\text{a) } c^2 = \vec{c} \cdot \vec{c} = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) = a^2 + b^2 - 2\vec{a} \cdot \vec{b} = a^2 + b^2 - 2ab \cos \theta.$$

Therefore,

$$c = \sqrt{a^2 + b^2 - 2ab \cos \theta}.$$

$$\begin{aligned} \text{b) } s(s-a)(s-b)(s-c) &= \frac{(a+b+c)}{2} \times \frac{(-a+b+c)}{2} \times \frac{(a-b+c)}{2} \times \frac{(a+b-c)}{2} \\ &= \frac{(a+b)^2 - c^2}{4} \times \frac{c^2 - (a-b)^2}{4} \\ &= \frac{(a^2 + b^2 + 2ab) - (a^2 + b^2 - 2ab \cos \theta)}{4} \times \frac{(a^2 + b^2 - 2ab \cos \theta) - (a^2 + b^2 - 2ab)}{4} \\ &= \frac{2ab(1 + \cos \theta)}{4} \times \frac{2ab(1 - \cos \theta)}{4} \\ &= \frac{a^2 b^2 (1 - \cos^2 \theta)}{4} \\ &= \left(\frac{ab \sin \theta}{2} \right)^2 \\ &= A^2. \end{aligned}$$
