

Problem 18) For $n = 4$, it is easy to see that $n! = 4! = 24 > 16 = 2^4 = 2^n$. Proof by induction now requires that we begin by assuming that $n!$ is greater than 2^n for some value of n that is greater than or equal to 4, then proceed to demonstrate that $(n + 1)!$ is greater than 2^{n+1} . We thus write

since, by assumption, $n \geq 4$, we have $n + 1 > 2$

$$(n + 1)! = n! \times (n + 1) > 2^n \times (n + 1) \overset{\downarrow}{>} 2^n \times 2 = 2^{n+1}.$$

The proof is now complete.
