Summer 2026 Written Comprehensive Exam
Opti 501, day 1

System of units: MKSA (also known as SI)

A plane wave having wave-vector $\boldsymbol{k}$ and frequency ω propagates in free space. Let the electric and magnetic fields of the plane-wave be written as

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_0 \exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)],$$

$$\boldsymbol{H}(\boldsymbol{r},t) = \boldsymbol{H}_0 \exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)].$$

No sources are assumed to reside in free space; therefore, $\rho_{\text{free}}(\boldsymbol{r},t) = 0$, $\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t) = 0$, $\boldsymbol{P}(\boldsymbol{r},t) = 0$, and $\boldsymbol{M}(\boldsymbol{r},t) = 0$.

2 Pts a) What does Maxwell's *first* equation (Gauss's law) have to say about the relation between $\boldsymbol{E}_0$ and $\boldsymbol{k}$?

2 Pts b) What does Maxwell's *fourth* equation (no magnetic monopoles) have to say about the relation between $\boldsymbol{H}_0$ and $\boldsymbol{k}$?

2 Pts c) Apply Maxwell's *second* equation (generalized Ampere's law) to derive a relation connecting $\boldsymbol{E}_0$, $\boldsymbol{H}_0$, $\boldsymbol{k}$, and ω.

2 Pts d) Apply Maxwell's *third* equation (Faraday's law) to derive another relation connecting $\boldsymbol{E}_0$, $\boldsymbol{H}_0$, $\boldsymbol{k}$, and ω.

2 Pts e) Combine the results obtained in parts (a)-(d) to eliminate $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ from the equations, thus arriving at the connection between $\boldsymbol{k}$ and ω for plane-waves that travel in free space.

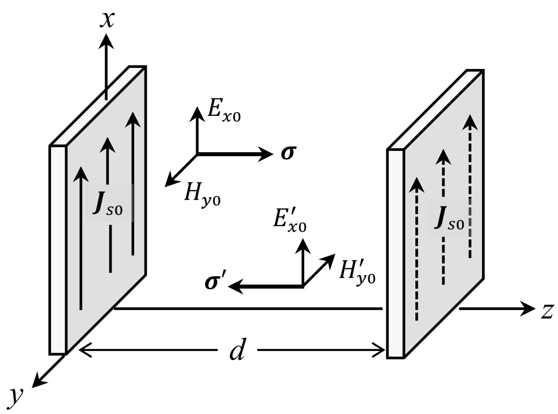
Hint: In part (e), the following vector identity will be helpful: $\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C}) = (\boldsymbol{A}\cdot\boldsymbol{C})\boldsymbol{B} - (\boldsymbol{A}\cdot\boldsymbol{B})\boldsymbol{C}$.

Summer 2026 Written Comprehensive Exam

Opti 501, Day 2

<u>System of units: SI (or MKSA)</u>

Two counter-propagating, linearly-polarized, homogeneous plane waves, both having frequency ω and wavelength λ_0, are trapped in the free-space region between a pair of perfectly conducting mirrors, as shown. The mirrors are parallel to the xy-plane, their separation d being an integer-multiple of $\lambda_0/2$. The propagation directions are $\boldsymbol{\sigma} = \hat{\boldsymbol{z}}$ and $\boldsymbol{\sigma}' = -\hat{\boldsymbol{z}}$. The (complex) E-field amplitudes $E_{x0} = |E_{x0}| \exp(\mathrm{i}\varphi_0)$ and $E'_{x0} = |E'_{x0}| \exp(\mathrm{i}\varphi'_0)$ have equal magnitudes; that is, $|E_{x0}| = |E'_{x0}|$.



3 Pts a) Write expressions for the total E- and H-fields in the cavity between the mirrors. (These expressions can be further simplified by taking into account the conditions at the mirrors.)

2 Pts b) In terms of $|E_{x0}|$, find the magnitude of the surface-current-density $J_{so}\hat{\boldsymbol{x}}$ on the mirror surfaces.

3 Pts c) Determine the total E- and H-field energies trapped within the cavity. Show that, at those instants when the E-field energy is zero, the H-field energy is at a maximum, and vice-versa.

2 Pts d) Write an expression for the Poynting vector $\boldsymbol{S}(z,t)$ inside the cavity. Interpret the oscillations of $\boldsymbol{S}(z,t)$ at a fixed location in space as time varies during each oscillation period.

Hint: $\cos a + \cos b = 2\cos[½(a+b)]\cos[½(a-b)]$;

$\cos a - \cos b = -2\sin[½(a+b)]\sin[½(a-b)]$.