Problem 1) a) The two E-fields are linearly polarized along the y-axis and co-propagate along the x-axis. Noting that $E_{y1} = E_s + E_d$ and $E_{y2} = E_s - E_d$, we write

$$\boldsymbol{E}(\boldsymbol{r},t) = E_{y1}\cos[(n_1\omega_1/c)x - \omega_1 t + \varphi_1]\,\hat{\boldsymbol{y}} + E_{y2}\cos[(n_2\omega_2/c)x - \omega_2 t + \varphi_2]\,\hat{\boldsymbol{y}}$$

$$= \{(E_s + E_d)\cos[(n_1\omega_1/c)x - \omega_1 t + \varphi_1] + (E_s - E_d)\cos[(n_2\omega_2/c)x - \omega_2 t + \varphi_2]\}\hat{\boldsymbol{y}}.$$

b) Using the trigonometric identities, we combine the two fields with amplitudes E_s, and also the two fields with amplitudes E_d, to arrive at

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{r},t) = {} & 2E_s\cos[\tfrac{1}{2}(\omega_1 n_1 + \omega_2 n_2)(x/c) - \tfrac{1}{2}(\omega_1 + \omega_2)t + \tfrac{1}{2}(\varphi_1 + \varphi_2)] \\ & \times \cos[\tfrac{1}{2}(\omega_1 n_1 - \omega_2 n_2)(x/c) - \tfrac{1}{2}(\omega_1 - \omega_2)t + \tfrac{1}{2}(\varphi_1 - \varphi_2)]\,\hat{\boldsymbol{y}} \\ & -2E_d\sin[\tfrac{1}{2}(\omega_1 n_1 + \omega_2 n_2)(x/c) - \tfrac{1}{2}(\omega_1 + \omega_2)t + \tfrac{1}{2}(\varphi_1 + \varphi_2)] \\ & \times \sin[\tfrac{1}{2}(\omega_1 n_1 - \omega_2 n_2)(x/c) - \tfrac{1}{2}(\omega_1 - \omega_2)t + \tfrac{1}{2}(\varphi_1 - \varphi_2)]\,\hat{\boldsymbol{y}} \\ \cong {} & (E_{y1} + E_{y2})\cos[(\omega_c n_c/c)x - \omega_c t + \tfrac{1}{2}(\varphi_1 + \varphi_2)] \\ & \times \cos\big[-\tfrac{1}{2}n_g(\omega_c)(\Delta\omega)(x/c) + \tfrac{1}{2}(\Delta\omega)t + \tfrac{1}{2}(\varphi_1 - \varphi_2)\big]\,\hat{\boldsymbol{y}} \\ & -(E_{y1} - E_{y2})\sin[(\omega_c n_c/c)x - \omega_c t + \tfrac{1}{2}(\varphi_1 + \varphi_2)] \\ & \times \sin\big[-\tfrac{1}{2}n_g(\omega_c)(\Delta\omega)(x/c) + \tfrac{1}{2}(\Delta\omega)t + \tfrac{1}{2}(\varphi_1 - \varphi_2)\big]\,\hat{\boldsymbol{y}}.\end{aligned}$$

c) The pair of beat signals may be further streamlined, as follows:

$$\boldsymbol{E}(\boldsymbol{r},t) = (E_{y1} + E_{y2})\cos\{(\omega_c n_c/c)[x - (c/n_c)t] + \tfrac{1}{2}(\varphi_1 + \varphi_2)\}$$

← carrier 1: frequency = ω_c, phase velocity = $c/n(\omega_c)$.

$$\times \cos\{\tfrac{1}{2}n_g(\omega_c)(\Delta\omega/c)[x - (c/n_g)t] - \tfrac{1}{2}(\varphi_1 - \varphi_2)\}\,\hat{\boldsymbol{y}}$$

← envelope 1: frequency = $\tfrac{1}{2}\Delta\omega$, group velocity = $c/n_g(\omega_c)$, amplitude = $E_{y1} + E_{y2}$.

$$+(E_{y1} - E_{y2})\sin\{(\omega_c n_c/c)[x - (c/n_c)t] + \tfrac{1}{2}(\varphi_1 + \varphi_2)\}$$

← carrier 2: frequency = ω_c, phase velocity = $c/n(\omega_c)$.

$$\times \sin\{\tfrac{1}{2}n_g(\omega_c)(\Delta\omega/c)[x - (c/n_g)t] - \tfrac{1}{2}(\varphi_1 - \varphi_2)\}\,\hat{\boldsymbol{y}}.$$

← envelope 2: frequency = $\tfrac{1}{2}\Delta\omega$, group velocity = $c/n_g(\omega_c)$, amplitude = $E_{y1} - E_{y2}$.

Note that the peak envelope of the first beat signal coincides with the zero of the envelope of the second beat signal and vice-versa. Also, in the special case where $E_{y1} = E_{y2}$, the second beat signal disappears.

Problem 2) a) At normal incidence, $k_x^{(\text{i})} = k_y^{(\text{i})} = 0$. Application of the generalized Snell's law now yields $\omega^{(\text{r})} = \omega^{(\text{t})} = \omega^{(\text{i})}$ and $k_x^{(\text{r})} = k_x^{(\text{t})} = 0$, as well as $k_y^{(\text{r})} = k_y^{(\text{t})} = 0$. From the dispersion relation, $\boldsymbol{k}\cdot\boldsymbol{k} = (\omega/c)^2\mu(\omega)\varepsilon(\omega)$, we find

$$k_z = \pm[(\omega/c)^2\mu(\omega)\varepsilon(\omega) - k_x^2 - k_y^2]^{1/2} = \pm(\omega/c)\sqrt{\mu(\omega)\varepsilon(\omega)}, \qquad (1)$$

where the plus or minus signs for $k_z^{(\text{i})}$, $k_z^{(\text{r})}$, and $k_z^{(\text{t})}$ must be chosen judiciously. We will have

$$k_z^{(\text{i})} = -(\omega/c)\sqrt{\mu_a(\omega)\varepsilon_a(\omega)} = -n_a\omega/c, \qquad (2)$$

$$k_z^{(\mathrm{r})} = (\omega/c)\sqrt{\mu_a(\omega)\varepsilon_a(\omega)} = n_a\omega/c, \tag{3}$$

understanding that $n_b'' > 0$

$$k_z^{(\mathrm{t})} = -(\omega/c)\sqrt{\mu_b(\omega)\varepsilon_b(\omega)} = -(n_b' + \mathrm{i}n_b'')\omega/c. \tag{4}$$

b) $\boldsymbol{\nabla}\cdot\boldsymbol{D}(\boldsymbol{r},t) = \rho_{\text{free}}(\boldsymbol{r},t) = 0 \quad \rightarrow \quad \mathrm{i}\boldsymbol{k}\cdot\varepsilon_0\varepsilon(\omega)\boldsymbol{E}_0 e^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} = 0 \quad \rightarrow \quad \boldsymbol{k}\cdot\boldsymbol{E}_0 = 0$

$$\rightarrow \quad \cancel{k_x}E_{0x} + \cancel{k_y}E_{0y} + k_z E_{0z} = 0 \quad \rightarrow \quad E_{0z}^{(\mathrm{i})} = E_{0z}^{(\mathrm{r})} = E_{0z}^{(\mathrm{t})} = 0. \tag{5}$$

c) $\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r},t) = -\partial\boldsymbol{B}(\boldsymbol{r},t)/\partial t \quad \rightarrow \quad \mathrm{i}\boldsymbol{k}\times\boldsymbol{E}_0 e^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} = \mathrm{i}\omega\mu_0\mu(\omega)\boldsymbol{H}_0 e^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$

$$\rightarrow \quad (\cancel{k_x}\hat{\boldsymbol{x}} + \cancel{k_y}\hat{\boldsymbol{y}} + k_z\hat{\boldsymbol{z}}) \times (E_{0x}\hat{\boldsymbol{x}} + E_{0y}\hat{\boldsymbol{y}} + \cancel{E_{0z}}\hat{\boldsymbol{z}}) = \omega\mu_0\mu(\omega)\boldsymbol{H}_0$$

$$\rightarrow \quad \boldsymbol{H}_0 = -(\omega\mu_0\mu)^{-1}k_z(E_{0y}\hat{\boldsymbol{x}} - E_{0x}\hat{\boldsymbol{y}}). \tag{6}$$

Recalling that $E_{0y}^{(\mathrm{i})} = 0$, we arrive at

$n_a = \sqrt{\mu_a\varepsilon_a}$, $c = (\mu_0\varepsilon_0)^{-½}$

$$\boldsymbol{H}_0^{(\mathrm{i})} = (\omega\mu_0\mu_a)^{-1}k_z^{(\mathrm{i})}E_{0x}^{(\mathrm{i})}\hat{\boldsymbol{y}} = -(n_a/c\mu_0\mu_a)E_{0x}^{(\mathrm{i})}\hat{\boldsymbol{y}} = -\sqrt{\frac{\varepsilon_0\varepsilon_a}{\mu_0\mu_a}}E_{0x}^{(\mathrm{i})}\hat{\boldsymbol{y}}. \tag{7}$$

$$\boldsymbol{H}_0^{(\mathrm{r})} = -(\omega\mu_0\mu_a)^{-1}k_z^{(\mathrm{r})}(E_{0y}^{(\mathrm{r})}\hat{\boldsymbol{x}} - E_{0x}^{(\mathrm{r})}\hat{\boldsymbol{y}}) = -\sqrt{\frac{\varepsilon_0\varepsilon_a}{\mu_0\mu_a}}(E_{0y}^{(\mathrm{r})}\hat{\boldsymbol{x}} - E_{0x}^{(\mathrm{r})}\hat{\boldsymbol{y}}). \tag{8}$$

$$\boldsymbol{H}_0^{(\mathrm{t})} = -(\omega\mu_0\mu_b)^{-1}k_z^{(\mathrm{t})}(E_{0y}^{(\mathrm{t})}\hat{\boldsymbol{x}} - E_{0x}^{(\mathrm{t})}\hat{\boldsymbol{y}}) = \sqrt{\frac{\varepsilon_0\varepsilon_b}{\mu_0\mu_b}}(E_{0y}^{(\mathrm{t})}\hat{\boldsymbol{x}} - E_{0x}^{(\mathrm{t})}\hat{\boldsymbol{y}}). \tag{9}$$

d) The continuity conditions for E_y and H_x fields at the interface (i.e., at $z = 0^{\pm}$) now become

$$\cancel{E_{0y}^{(\mathrm{i})}} + E_{0y}^{(\mathrm{r})} = E_{0y}^{(\mathrm{t})} \quad \rightarrow \quad E_{0y}^{(\mathrm{r})} = E_{0y}^{(\mathrm{t})}, \tag{10}$$

$$\cancel{H_{0x}^{(\mathrm{i})}} + H_{0x}^{(\mathrm{r})} = H_{0x}^{(\mathrm{t})} \quad \rightarrow \quad -\sqrt{\frac{\varepsilon_0\varepsilon_a}{\mu_0\mu_a}}E_{0y}^{(\mathrm{r})} = \sqrt{\frac{\varepsilon_0\varepsilon_b}{\mu_0\mu_b}}E_{0y}^{(\mathrm{t})}. \tag{11}$$

The only solution of Eqs.(10) and (11) is $E_{0y}^{(\mathrm{r})} = E_{0y}^{(\mathrm{t})} = 0$. The remaining boundary conditions (i.e., those pertaining to the continuity of E_x and H_y at $z = 0^{\pm}$) yield

$$E_{0x}^{(\mathrm{i})} + E_{0x}^{(\mathrm{r})} = E_{0x}^{(\mathrm{t})}, \tag{12}$$

$$H_{0y}^{(\mathrm{i})} + H_{0y}^{(\mathrm{r})} = H_{0y}^{(\mathrm{t})} \quad \rightarrow \quad -\sqrt{\frac{\varepsilon_0\varepsilon_a}{\mu_0\mu_a}}E_{0x}^{(\mathrm{i})} + \sqrt{\frac{\varepsilon_0\varepsilon_a}{\mu_0\mu_a}}E_{0x}^{(\mathrm{r})} = -\sqrt{\frac{\varepsilon_0\varepsilon_b}{\mu_0\mu_b}}E_{0x}^{(\mathrm{t})}$$

$$\rightarrow \quad \sqrt{\varepsilon_a/\mu_a}\,(E_{0x}^{(\mathrm{i})} - E_{0x}^{(\mathrm{r})}) = \sqrt{\varepsilon_b/\mu_b}\,E_{0x}^{(\mathrm{t})}. \tag{13}$$

Substituting from Eq.(12) into Eq.(13), we arrive at

$$\sqrt{\varepsilon_a/\mu_a}\,(E_{0x}^{(\mathrm{i})} - E_{0x}^{(\mathrm{r})}) = \sqrt{\varepsilon_b/\mu_b}\,(E_{0x}^{(\mathrm{i})} + E_{0x}^{(\mathrm{r})}). \tag{14}$$

Solving the above equations, we find

$$\rho = E_{0x}^{(\mathrm{r})}/E_{0x}^{(\mathrm{i})} = \frac{\sqrt{\varepsilon_a/\mu_a} - \sqrt{\varepsilon_b/\mu_b}}{\sqrt{\varepsilon_a/\mu_a} + \sqrt{\varepsilon_b/\mu_b}}, \tag{15}$$

$$\tau = E_{0x}^{(\mathrm{t})}/E_{0x}^{(\mathrm{i})} = \frac{2\sqrt{\varepsilon_a/\mu_a}}{\sqrt{\varepsilon_a/\mu_a} + \sqrt{\varepsilon_b/\mu_b}}. \tag{16}$$

e) The reflected and transmitted plane-waves consist of the following $\boldsymbol{E}$ and $\boldsymbol{H}$ fields:

$$\boldsymbol{E}^{(\mathrm{r})}(\boldsymbol{r},t)=\rho E_{0x}^{(\mathrm{i})}e^{\mathrm{i}(k_z^{(\mathrm{r})}z-\omega t)}\hat{\boldsymbol{x}}, \quad (17)$$

$$\boldsymbol{E}^{(\mathrm{t})}(\boldsymbol{r},t)=\tau E_{0x}^{(\mathrm{i})}e^{\mathrm{i}(k_z^{(\mathrm{t})}z-\omega t)}\hat{\boldsymbol{x}}, \quad (18)$$

$$\boldsymbol{H}^{(\mathrm{r})}(\boldsymbol{r},t)=H_{0y}^{(\mathrm{r})}e^{\mathrm{i}(k_z^{(\mathrm{r})}z-\omega t)}\hat{\boldsymbol{y}}=\rho\sqrt{\varepsilon_0\varepsilon_a/\mu_0\mu_a}\,E_{0x}^{(\mathrm{i})}e^{\mathrm{i}(k_z^{(\mathrm{r})}z-\omega t)}\hat{\boldsymbol{y}}, \quad \leftarrow \text{see Eq.(8)} \quad (19)$$

$$\boldsymbol{H}^{(\mathrm{t})}(\boldsymbol{r},t)=H_{0y}^{(\mathrm{t})}e^{\mathrm{i}(k_z^{(\mathrm{t})}z-\omega t)}\hat{\boldsymbol{y}}=(\omega\mu_0\mu_b)^{-1}k_z^{(\mathrm{t})}E_{0x}^{(\mathrm{t})}e^{\mathrm{i}(k_z^{(\mathrm{t})}z-\omega t)}\hat{\boldsymbol{y}}. \quad \leftarrow \text{see Eq.(6)} \quad (20)$$

The above expression of $\boldsymbol{H}^{(\mathrm{t})}$ can be further simplified, but Eq.(20) is convenient for use in part (f).

f) In the limit when $\mu_b\to\infty$, we will have $\sqrt{\varepsilon_b/\mu_b}\to 0$, in which case $\rho\to 1$ and $\tau\to 2$; see Eqs.(15) and (16). The tangential H-field immediately above the interface, namely, $H_{0y}^{(\mathrm{i})}+H_{0y}^{(\mathrm{r})}$, now approaches $-\sqrt{\varepsilon_0\varepsilon_a/\mu_0\mu_a}\,(1-\rho)E_{0x}^{(\mathrm{i})}=0$; see Eqs.(7) and (19). Inside the transmittance medium b, the H-field drops to zero everywhere due to the rapid exponential decay of $e^{\mathrm{i}k_z^{(t)}z}$; see Eq.(4) — also, in accordance with Eq.(9), $H_{0y}^{(\mathrm{t})}\to 0$ as $\mu_b\to\infty$. The tangential H-field thus remains continuous and a surface-electric-current does *not* appear in the system.

The situation is markedly different for $\boldsymbol{E}_{\parallel}$ at the $z=0$ interface as $\mu_b\to\infty$ (and, consequently, $\rho\to 1$ and $\tau\to 2$). Here, $\boldsymbol{E}_{\parallel}$ immediately above the interface will be $E_{0x}^{(\mathrm{i})}+E_{0x}^{(\mathrm{r})}=(1+\rho)E_{0x}^{(\mathrm{i})}\to 2E_{0x}^{(\mathrm{i})}$. However, inside the transmittance medium b, the E-field everywhere approaches zero due to the rapid exponential decline of $e^{\mathrm{i}k_z^{(\mathrm{t})}z}$ for $z<0$. Considering that $\boldsymbol{B}^{(\mathrm{t})}(\boldsymbol{r},t)=\mu_0\mu_b\boldsymbol{H}^{(\mathrm{t})}(\boldsymbol{r},t)$, Eq.(20) yields

$$\partial B_y^{(\mathrm{t})}(\boldsymbol{r},t)/\partial t=-\mathrm{i}\omega\mu_0\mu_b H_y^{(\mathrm{t})}(\boldsymbol{r},t)=-\mathrm{i}k_z^{(\mathrm{t})}E_{0x}^{(\mathrm{t})}e^{\mathrm{i}(k_z^{(\mathrm{t})}z-\omega t)}. \quad (21)$$

Integrating the above expression over the entire penetration depth of the transmitted B-field, one arrives at

$$\int_{z=-\infty}^{0}[\partial B_y^{(\mathrm{t})}(\boldsymbol{r},t)/\partial t]\mathrm{d}z=-E_{0x}^{(\mathrm{t})}e^{\mathrm{i}(k_z^{(\mathrm{t})}z-\omega t)}\Big|_{z=-\infty}^{0}=-E_{0x}^{(\mathrm{t})}e^{-\mathrm{i}\omega t}=-\tau E_{0x}^{(\mathrm{i})}e^{-\mathrm{i}\omega t}. \quad (22)$$

Thus, in the limit when $\mu_b\to\infty$, while $\partial\boldsymbol{B}^{(\mathrm{t})}/\partial t$ goes to zero everywhere that z is negative, the integral of $\partial B_y^{(\mathrm{t})}/\partial t$ over the entire depth of medium b approaches $-2E_{0x}^{(\mathrm{i})}e^{-\mathrm{i}\omega t}$; see Eq.(22). This, of course, is consistent with the boundary condition according to Maxwell's third equation, $\boldsymbol{\nabla}\times\boldsymbol{E}=-\partial\boldsymbol{B}/\partial t$, because the time-derivative of the magnetic induction $\boldsymbol{B}$, now confined to the surface of medium b and acting as a surface-magnetic-current-density (directed along the y-axis), accounts for the discontinuity of E_x at the $z=0$ interface.

Problem 3) a) According to the generalized Snell's law, $\omega^{(\mathrm{i})}=\omega^{(\mathrm{r})}=\omega^{(\mathrm{t})}$ and $k_x^{(\mathrm{i})}=k_x^{(\mathrm{r})}=k_x^{(\mathrm{t})}$; also, $k_y^{(\mathrm{i})}=k_y^{(\mathrm{r})}=k_y^{(\mathrm{t})}=0$. Therefore, $k_z^{(\mathrm{r})}=\sqrt{(\omega/c)^2n_a^2-k_x^2}=(\omega/c)n_a\cos\theta$. Note that the chosen sign for $k_z^{(\mathrm{r})}$ is positive, ensuring that the reflected beam propagates upward, along the z-axis. As for the transmitted beam, the dispersion relation yields $k_z^{(\mathrm{t})}=\pm\sqrt{(\omega/c)^2n_b^2-k_x^2}$. The sign of $k_z^{(\mathrm{t})}$ must be chosen to ensure the exponential decay of the transmitted wave along the negative z-axis; in other words, the imaginary part of $k_z^{(\mathrm{t})}$ must be negative. With this understanding, we proceed to use the minus sign for $k_z^{(\mathrm{t})}$ in the equations that follow.

b) $\boldsymbol{\nabla}\cdot\boldsymbol{D}(\boldsymbol{r},t)=\rho_{\text{free}}(\boldsymbol{r},t)=0 \quad\rightarrow\quad \mathrm{i}\boldsymbol{k}\cdot\varepsilon_0\varepsilon(\omega)\boldsymbol{E}_0 e^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}=0 \quad\rightarrow\quad \boldsymbol{k}\cdot\boldsymbol{E}_0=0$

$$\rightarrow\quad k_x E_{0x}+\cancel{k_y}E_{0y}+k_z E_{0z}=0 \quad\rightarrow\quad E_{0z}=-k_x E_{0x}/k_z. \tag{1}$$

The above identity holds for all three plane-waves. Therefore,

$$E_{0z}^{(\mathrm{i})}=-\frac{(\omega/c)n_a\sin\theta E_{0x}^{(\mathrm{i})}}{-(\omega/c)n_a\cos\theta}=(\tan\theta)E_{0x}^{(\mathrm{i})}, \tag{2}$$

$$E_{0z}^{(\mathrm{r})}=-\frac{(\omega/c)n_a\sin\theta E_{0x}^{(\mathrm{r})}}{(\omega/c)n_a\cos\theta}=-(\tan\theta)E_{0x}^{(\mathrm{r})}, \tag{3}$$

$$E_{0z}^{(\mathrm{t})}=-\frac{(\omega/c)n_a\sin\theta E_{0x}^{(\mathrm{t})}}{-[(\omega/c)^2n_b^2-k_x^2]^{½}}=\frac{n_a\sin\theta}{[n_b^2-n_a^2\sin^2\theta]^{½}}E_{0x}^{(\mathrm{t})}. \tag{4}$$

c) $\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r},t)=-\partial\boldsymbol{B}(\boldsymbol{r},t)/\partial t \quad\rightarrow\quad \mathrm{i}\boldsymbol{k}\times\boldsymbol{E}_0 e^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}=\mathrm{i}\omega\mu_0\mu(\omega)\boldsymbol{H}_0 e^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$

$$\rightarrow\quad (k_x\hat{\boldsymbol{x}}+\cancel{k_y}\hat{\boldsymbol{y}}+k_z\hat{\boldsymbol{z}})\times(E_{0x}\hat{\boldsymbol{x}}+E_{0y}\hat{\boldsymbol{y}}+E_{0z}\hat{\boldsymbol{z}})=\omega\mu_0\mu(\omega)\boldsymbol{H}_0$$

$$\rightarrow\quad \boldsymbol{H}_0=(\omega\mu_0\mu)^{-1}[-k_zE_{0y}\hat{\boldsymbol{x}}+(k_zE_{0x}-k_xE_{0z})\hat{\boldsymbol{y}}+k_xE_{0y}\hat{\boldsymbol{z}}]$$

invoking Eq.(1)

$$\rightarrow\quad \boldsymbol{H}_0=(\omega\mu_0\mu)^{-1}\left[-k_zE_{0y}\hat{\boldsymbol{x}}+\left(\frac{k_x^2+k_z^2}{k_z}\right)E_{0x}\hat{\boldsymbol{y}}+k_xE_{0y}\hat{\boldsymbol{z}}\right]. \tag{5}$$

Substituting $(\omega/c)n_a\sin\theta$ for k_x and also the relevant expressions for $k_z^{(\mathrm{i})}$, $k_z^{(\mathrm{r})}$ and $k_z^{(\mathrm{t})}$, we find

$$\boldsymbol{H}_0^{(\mathrm{i})}=\sqrt{\varepsilon_0\varepsilon_a/\mu_0\mu_a}\,[\cos\theta\,E_{0y}^{(\mathrm{i})}\hat{\boldsymbol{x}}-(E_{0x}^{(\mathrm{i})}/\cos\theta)\hat{\boldsymbol{y}}+\sin\theta\,E_{0y}^{(\mathrm{i})}\hat{\boldsymbol{z}}], \tag{6}$$

$$\boldsymbol{H}_0^{(\mathrm{r})}=\sqrt{\varepsilon_0\varepsilon_a/\mu_0\mu_a}\,[-\cos\theta\,E_{0y}^{(\mathrm{r})}\hat{\boldsymbol{x}}+(E_{0x}^{(\mathrm{r})}/\cos\theta)\hat{\boldsymbol{y}}+\sin\theta\,E_{0y}^{(\mathrm{r})}\hat{\boldsymbol{z}}], \tag{7}$$

$$\boldsymbol{H}_0^{(\mathrm{t})}=(c\mu_0\mu_b)^{-1}[\sqrt{n_b^2-n_a^2\sin^2\theta}\,E_{0y}^{(\mathrm{t})}\hat{\boldsymbol{x}}-\frac{n_b^2}{(n_b^2-n_a^2\sin^2\theta)^{½}}E_{0x}^{(\mathrm{t})}\hat{\boldsymbol{y}}+n_a\sin\theta\,E_{0y}^{(\mathrm{t})}\hat{\boldsymbol{z}}]. \tag{8}$$

d) The continuity of E_x, E_y, H_x and H_y at the $z=0$ boundary between media a and b yields

i) $E_{0x}^{(\mathrm{i})}+E_{0x}^{(\mathrm{r})}=E_{0x}^{(\mathrm{t})},$ (9)

ii) $H_{0y}^{(\mathrm{i})}+H_{0y}^{(\mathrm{r})}=H_{0y}^{(\mathrm{t})} \quad\rightarrow\quad \sqrt{\frac{\varepsilon_a}{\mu_a}}\left(\frac{E_{0x}^{(\mathrm{i})}}{\cos\theta}-\frac{E_{0x}^{(\mathrm{r})}}{\cos\theta}\right)=\frac{n_b^2E_{0x}^{(\mathrm{t})}}{\mu_b(n_b^2-n_a^2\sin^2\theta)^{½}}$

$$\rightarrow\quad E_{0x}^{(\mathrm{i})}-E_{0x}^{(\mathrm{r})}=\frac{\varepsilon_bn_a\cos\theta}{\varepsilon_a(n_b^2-n_a^2\sin^2\theta)^{½}}E_{0x}^{(\mathrm{t})}, \tag{10}$$

iii) $E_{0y}^{(\mathrm{i})}+E_{0y}^{(\mathrm{r})}=E_{0y}^{(\mathrm{t})},$ (11)

iv) $H_{0x}^{(\mathrm{i})}+H_{0x}^{(\mathrm{r})}=H_{0x}^{(\mathrm{t})} \quad\rightarrow\quad \sqrt{\varepsilon_a/\mu_a}\,(\cos\theta\,E_{0y}^{(\mathrm{i})}-\cos\theta\,E_{0y}^{(\mathrm{r})})=(1/\mu_b)\sqrt{n_b^2-n_a^2\sin^2\theta}\,E_{0y}^{(\mathrm{t})}$

$$\rightarrow\quad E_{0y}^{(\mathrm{i})}-E_{0y}^{(\mathrm{r})}=\frac{\mu_a(n_b^2-n_a^2\sin^2\theta)^{½}}{\mu_bn_a\cos\theta}E_{0y}^{(\mathrm{t})}. \tag{12}$$

Equations (9) and (10) are a pair of coupled equations that can be solved for the Fresnel reflection and transmission coefficients for p-polarized incident light, namely, $\rho_p=E_{0x}^{(\mathrm{r})}/E_{0x}^{(\mathrm{i})}$ and $\tau_p=E_{0x}^{(\mathrm{t})}/E_{0x}^{(\mathrm{i})}$. Similarly, Eqs.(11) and (12) are a coupled pair that can be solved for $\rho_s=E_{0y}^{(\mathrm{r})}/E_{0y}^{(\mathrm{i})}$ and $\tau_s=E_{0y}^{(\mathrm{t})}/E_{0y}^{(\mathrm{i})}$ (i.e., the Fresnel reflection and transmission coefficients for s-polarized light.)
