Solution to Problem 2) a) $\boldsymbol{E}(\boldsymbol{r},t)=\boldsymbol{E}_0\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]$, $\boldsymbol{H}(\boldsymbol{r},t)=\boldsymbol{H}_0\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]$.

b) i) $\boldsymbol{\nabla}\cdot\boldsymbol{D}(\boldsymbol{r},t)=\rho_{\text{free}}(\boldsymbol{r},t) \;\rightarrow\; \mathrm{i}\boldsymbol{k}\cdot\varepsilon_0\varepsilon(\omega)\boldsymbol{E}_0\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]=0 \;\rightarrow\; \boldsymbol{k}\cdot\boldsymbol{E}_0=0.$

ii) $\boldsymbol{\nabla}\times\boldsymbol{H}(\boldsymbol{r},t)=\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t)+\frac{\partial\boldsymbol{D}(\boldsymbol{r},t)}{\partial t}$

$\rightarrow\; \mathrm{i}\boldsymbol{k}\times\boldsymbol{H}_0\cancel{e^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}}=-\mathrm{i}\omega\varepsilon_0\varepsilon(\omega)\boldsymbol{E}_0\cancel{e^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}} \;\rightarrow\; \boldsymbol{k}\times\boldsymbol{H}_0=-\varepsilon_0\varepsilon(\omega)\omega\boldsymbol{E}_0.$

iii) $\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r},t)=-\frac{\partial\boldsymbol{B}(\boldsymbol{r},t)}{\partial t}$

$\rightarrow\; \mathrm{i}\boldsymbol{k}\times\boldsymbol{E}_0\cancel{e^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}}=\mathrm{i}\omega\mu_0\mu(\omega)\boldsymbol{H}_0\cancel{e^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}} \;\rightarrow\; \boldsymbol{k}\times\boldsymbol{E}_0=\mu_0\mu(\omega)\omega\boldsymbol{H}_0.$

iv) $\boldsymbol{\nabla}\cdot\boldsymbol{B}(\boldsymbol{r},t)=0 \;\rightarrow\; \mathrm{i}\boldsymbol{k}\cdot\mu_0\mu(\omega)\boldsymbol{H}_0\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]=0 \;\rightarrow\; \boldsymbol{k}\cdot\boldsymbol{H}_0=0.$

c) $\boldsymbol{k}\cdot\boldsymbol{E}_0=0 \;\rightarrow\; k_xE_{0x}+k_yE_{0y}+k_zE_{0z}=0 \;\rightarrow\; E_{0z}=-(k_xE_{0x}+k_yE_{0y})/k_z.$

d) Multiply both sides of Eq.(ii) into $\mu_0\mu(\omega)\omega$, then substitute $\boldsymbol{k}\times\boldsymbol{E}_0$ for $\mu_0\mu(\omega)\omega\boldsymbol{H}_0$ from Eq.(iii) to arrive at

from Eq.(i) 0

$$\boldsymbol{k}\times(\boldsymbol{k}\times\boldsymbol{E}_0)=-\mu_0\varepsilon_0\omega^2\mu(\omega)\varepsilon(\omega)\boldsymbol{E}_0 \;\rightarrow\; \cancel{(\boldsymbol{k}\cdot\boldsymbol{E}_0)\boldsymbol{k}}-(\boldsymbol{k}\cdot\boldsymbol{k})\cancel{\boldsymbol{E}_0}=-(\omega/c)^2\mu(\omega)\varepsilon(\omega)\cancel{\boldsymbol{E}_0}$$

$$\rightarrow\; \boldsymbol{k}\cdot\boldsymbol{k}=(\omega/c)^2\mu(\omega)\varepsilon(\omega).$$

Traditionally, the refractive index is defined as $n(\omega)=\sqrt{\mu(\omega)\varepsilon(\omega)}$. Consequently, the above dispersion relation may equivalently be written as $k^2=[n(\omega)\omega/c]^2$.