PhD Qualifying Exam, Summer 2023

Opti 501, Day 2

System of units: SI (or MKSA)

a) For an electromagnetic plane-wave residing inside a linear, isotropic, and homogeneous medium, write a (generally complex) pair of expressions for the electric field $\boldsymbol{E}(\boldsymbol{r},t)$ and the magnetic field $\boldsymbol{H}(\boldsymbol{r},t)$ in terms of $\boldsymbol{E}_0, \boldsymbol{H}_0, \boldsymbol{k}$, and ω.

b) Assuming that $\rho_{\text{free}}(\boldsymbol{r},t) = 0$ and $\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t) = 0$, write all four equations of Maxwell for the E-field and H-field of the plane-wave, keeping in mind that the remaining sources within the medium should be expressed as $\boldsymbol{P}(\boldsymbol{r},t) = \varepsilon_0 \chi_{\text{e}}(\omega)\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{M}(\boldsymbol{r},t) = \mu_0 \chi_{\text{m}}(\omega)\boldsymbol{H}(\boldsymbol{r},t)$. Simplify these equations by eliminating the $\boldsymbol{\nabla}\cdot$, $\boldsymbol{\nabla}\times$, $\partial/\partial t$ operators and using the material medium's (relative) permittivity $\varepsilon(\omega) = 1 + \chi_{\text{e}}(\omega)$ and permeability $\mu(\omega) = 1 + \chi_{\text{m}}(\omega)$.

c) Use Maxwell's 1st equation obtained in part (b) to express E_{0z} in terms of $E_{0x}, E_{0y}, k_x, k_y, k_z$.

d) Derive the dispersion relation $\boldsymbol{k}\cdot\boldsymbol{k} = (\omega/c)^2\,\mu(\omega)\varepsilon(\omega)$ from the Maxwell equations obtained in part (b). How is the material medium's (complex) refractive index $n(\omega)$ related to its (relative) permittivity $\varepsilon(\omega)$ and (relative) permeability $\mu(\omega)$?

Hint: The vector identity $\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c}) = (\boldsymbol{a}\cdot\boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a}\cdot\boldsymbol{b})\boldsymbol{c}$ should be helpful.
