Problem 1) a) N is the number of oscillating electrons per unit volume; its SI units are $[1/\mathrm{m}^3]$.

q is the effective charge of the oscillating particle (typically an electron); its units are [coulomb].

ε_0 is the permittivity of free space; its units are [farad/m].

m is the effective mass of the oscillating particle (typically an electron); its units are [kg].

α is the spring constant. The model assumes that a spring connects the oscillating particle to the atomic/molecular nucleus (or the underlying lattice). α is the proportionality coefficient between the restoring force acting on the particle and the particle's displacement from equilibrium. The SI units of α are [newton/m].

β, the friction coefficient, is the proportionality constant relating the overall frictional force acting on the oscillating particle to the particle's instantaneous velocity. The SI units of β are $[\text{newton} \cdot \text{sec/m}]$.

The plasma frequency ω_p, the resonance frequency ω_0, and the damping coefficient γ, all have the units of frequency, namely, [1/sec]. This should be clear from the way these parameters appear in the mathematical expression of $\chi_e(\omega)$.

b) Conduction electrons differ from bound electrons in that they are *not* connected to a nucleus (or to an underlying lattice) by a fictitious spring that would apply a restoring force to the electron. Therefore, for a conduction electron, the spring constant α is essentially zero, which makes the resonance frequency ω_0 equal to zero as well.

The Clausius-Mossotti correction is intended to remove the contribution of the local electric field $\boldsymbol{E}(\boldsymbol{r})e^{-\mathrm{i}\omega t}$ to the restoring force that acts on the oscillating particle — i.e., that part of the local E-field that is considered to be the self-field. This is because the Lorentz oscillator model incorporates an overall restoring force by allowing for a spring, whose spring constant is α. However, for conduction electrons, no such spring has been assumed and, therefore, there is no chance of double-counting the restoring force. Consequently, the Drude model of the conduction electron (i.e., the Lorentz oscillator model in which ω_0 is set to zero) has no need for correction.

c) Given $\boldsymbol{P}(\boldsymbol{r},t) = \varepsilon_0\chi_e(\omega)\boldsymbol{E}(\boldsymbol{r})e^{-\mathrm{i}\omega t}$, the electric current density will be

$$\boldsymbol{J}(\boldsymbol{r},t) = \partial\boldsymbol{P}/\partial t = -\mathrm{i}\omega\varepsilon_0\chi_e(\omega)\boldsymbol{E}(\boldsymbol{r})e^{-\mathrm{i}\omega t} \quad \rightarrow \quad \sigma(\omega) = -\mathrm{i}\omega\varepsilon_0\chi_e(\omega).$$

In the Drude model, we have $\omega_0 = 0$. Consequently, $\chi_e(\omega) = -\omega_p^2/(\omega^2 + \mathrm{i}\gamma\omega)$, which leads to

$$\sigma(\omega) = \mathrm{i}\varepsilon_0\omega_p^2/(\omega + \mathrm{i}\gamma) = (Nq^2/m)/(\gamma - \mathrm{i}\omega).$$

One can readily verify that the units of $\sigma(\omega)$ are [ampere/(volt · m)], i.e., the units of the current-density $[\text{ampere/m}^2]$ divided by those of the electric field [volt/m]. The electrical conductivity $\sigma(\omega)$ is related to electric resistance, whose units are [volt/ampere] or ohm [Ω]. Thus, the units of $\sigma(\omega)$ may also be described as $[1/(\Omega \cdot \mathrm{m})]$.

Problem 2) a) Considering that $c_2^* = c_2' - \mathrm{i}c_2''$, the real part of c_2 can be written as $½(c_2 + c_2^*)$. Therefore, $\mathrm{Re}(c_1)\mathrm{Re}(c_2) = \mathrm{Re}[c_1\mathrm{Re}(c_2)] = ½\mathrm{Re}[c_1(c_2 + c_2^*)]$.

b) $$\tilde{\boldsymbol{S}} = \mathrm{Re}(\boldsymbol{E} \times \boldsymbol{H}) = \mathrm{Re}[(\boldsymbol{E}' + \mathrm{i}\boldsymbol{E}'') \times (\boldsymbol{H}' + \mathrm{i}\boldsymbol{H}'')]$$

$$= \mathrm{Re}[(\boldsymbol{E}' \times \boldsymbol{H}' - \boldsymbol{E}'' \times \boldsymbol{H}'') + \mathrm{i}(\boldsymbol{E}' \times \boldsymbol{H}'' + \boldsymbol{E}'' \times \boldsymbol{H}')] = \boldsymbol{E}' \times \boldsymbol{H}' - \boldsymbol{E}'' \times \boldsymbol{H}''. \qquad (1)$$

Clearly, $\tilde{\boldsymbol{S}}$ differs from $\boldsymbol{S} = \text{Re}(\boldsymbol{E}) \times \text{Re}(\boldsymbol{H}) = \boldsymbol{E}' \times \boldsymbol{H}'$, because an additional term, $\boldsymbol{E}'' \times \boldsymbol{H}''$, appears in the above expression of $\tilde{\boldsymbol{S}}$.

c) $$\boldsymbol{S}(\boldsymbol{r},t) = \text{Re}(\boldsymbol{E}) \times \text{Re}(\boldsymbol{H}) = \text{Re}(\boldsymbol{E}) \times ½(\boldsymbol{H} + \boldsymbol{H}^*) = ½\text{Re}[\boldsymbol{E} \times (\boldsymbol{H} + \boldsymbol{H}^*)]$$
$$= ½\text{Re}(\boldsymbol{E} \times \boldsymbol{H} + \boldsymbol{E} \times \boldsymbol{H}^*) = ½\text{Re}(\boldsymbol{E} \times \boldsymbol{H}) + ½\text{Re}(\boldsymbol{E} \times \boldsymbol{H}^*). \tag{2}$$

d) $$\boldsymbol{S}(\boldsymbol{r},t) = ½\text{Re}\big[\boldsymbol{E}(\boldsymbol{r})e^{-\text{i}\omega t} \times \boldsymbol{H}(\boldsymbol{r})e^{-\text{i}\omega t}\big] + ½\text{Re}\big[\boldsymbol{E}(\boldsymbol{r})\cancel{e^{-\text{i}\omega t}} \times \boldsymbol{H}^*(\boldsymbol{r})\cancel{e^{+\text{i}\omega t}}\big]$$
$$= ½\text{Re}[\boldsymbol{E}(\boldsymbol{r}) \times \boldsymbol{H}(\boldsymbol{r})(\cos 2\omega t - \text{i}\sin 2\omega t)] + ½\text{Re}[\boldsymbol{E}(\boldsymbol{r}) \times \boldsymbol{H}^*(\boldsymbol{r})]$$
$$= ½\text{Re}[\boldsymbol{E}(\boldsymbol{r}) \times \boldsymbol{H}(\boldsymbol{r})]\cos(2\omega t) + ½\text{Im}[\boldsymbol{E}(\boldsymbol{r}) \times \boldsymbol{H}(\boldsymbol{r})]\sin(2\omega t)$$
$$+½\text{Re}[\boldsymbol{E}(\boldsymbol{r}) \times \boldsymbol{H}^*(\boldsymbol{r})]. \tag{3}$$

e) $$\langle\cos(2\omega_0 t)\rangle = T^{-1}\int_{t_0}^{t_0+T}\cos(2\omega_0 t)\,\text{d}t = (2\omega_0 T)^{-1}\sin(2\omega_0 t)\big|_{t=t_0}^{t_0+T}$$
$$= (2\omega_0 T)^{-1}[\cancel{\sin(2\omega_0 t_0 + 2\pi)} - \cancel{\sin(2\omega_0 t_0)}] = 0. \tag{4}$$

A similar calculation shows that $\langle\sin(2\omega_0 t)\rangle = 0$. Substitution into Eq.(3) now yields
$$\langle\boldsymbol{S}(\boldsymbol{r},t)\rangle = ½\text{Re}[\boldsymbol{E}(\boldsymbol{r}) \times \boldsymbol{H}^*(\boldsymbol{r})]. \tag{5}$$

Problem 3) a) From the generalized Snell's law, we have $\omega^{(\text{i})} = \omega^{(\text{r})} = \omega^{(\text{t})} = \omega$, $k_x^{(\text{i})} = k_x^{(\text{r})} = k_x^{(\text{t})} = k_x$ and $k_y^{(\text{i})} = k_y^{(\text{r})} = k_y^{(\text{t})} = k_y$. Since the plane of incidence is chosen to be the xz-plane, we have $k_y = 0$. Since the incident plane-wave is said to be homogeneous, the dispersion relation yields $|\boldsymbol{k}^{(\text{i})}| = n_0\omega/c$ and, therefore, $k_x = (n_0\omega/c)\sin\theta$. Since the incident plane-wave is downward propagating, we have $k_z^{(\text{i})} = -(n_0\omega/c)\cos\theta$. Similar arguments can be used to obtain the expressions of $k_z^{(\text{r})}$ and $k_z^{(\text{t})}$. Consequently,
$$\boldsymbol{k}^{(\text{i})} = (n_0\omega/c)(\sin\theta\,\hat{\boldsymbol{x}} - \cos\theta\,\hat{\boldsymbol{z}}), \tag{1}$$
$$\boldsymbol{k}^{(\text{r})} = (n_0\omega/c)(\sin\theta\,\hat{\boldsymbol{x}} + \cos\theta\,\hat{\boldsymbol{z}}), \tag{2}$$
$$\boldsymbol{k}^{(\text{t})} = (n_0\omega/c)\big(\sin\theta\,\hat{\boldsymbol{x}} - \sqrt{(n_1/n_0)^2 - \sin^2\theta}\,\hat{\boldsymbol{z}}\big). \tag{3}$$

The incident E-field is written as $\boldsymbol{E}^{(\text{i})}(\boldsymbol{r},t) = (E_{0x}^{(\text{i})}\hat{\boldsymbol{x}} + E_{0z}^{(\text{i})}\hat{\boldsymbol{z}})\exp[\text{i}(\boldsymbol{k}^{(\text{i})}\cdot\boldsymbol{r} - \omega t)]$. Maxwell's 1st equation now relates the z-component of the E-field to its x-component, as follows:
$$\boldsymbol{\nabla}\cdot\boldsymbol{E}^{(\text{i})}(\boldsymbol{r},t) = 0 \;\rightarrow\; \boldsymbol{k}^{(\text{i})}\cdot\boldsymbol{E}_0^{(\text{i})} = 0 \;\rightarrow\; k_x E_{0x}^{(\text{i})} + k_z^{(\text{i})}E_{0z}^{(\text{i})} = 0 \;\rightarrow\; E_{0z}^{(\text{i})} = (\tan\theta)E_{0x}^{(\text{i})}. \tag{4}$$

Similar expressions are found for $E_{0z}^{(\text{r})}$ and $E_{0z}^{(\text{t})}$; that is,
$$k_x E_{0x}^{(\text{r})} + k_z^{(\text{r})}E_{0z}^{(\text{r})} = 0 \quad\rightarrow\quad E_{0z}^{(\text{r})} = -(\tan\theta)E_{0x}^{(\text{r})}, \tag{5}$$
$$k_x E_{0x}^{(\text{t})} + k_z^{(\text{t})}E_{0z}^{(\text{t})} = 0 \quad\rightarrow\quad E_{0z}^{(\text{t})} = \frac{(\sin\theta)E_{0x}^{(\text{t})}}{\sqrt{(n_1/n_0)^2 - \sin^2\theta}}. \tag{6}$$

For each plane-wave, the magnetic field $\boldsymbol{H}(\boldsymbol{r},t)$ has only one component along the y-axis. This component can be found from Maxwell's 3rd equation, as follows:
$$\boldsymbol{\nabla}\times\boldsymbol{E} = -\partial\boldsymbol{B}/\partial t \;\rightarrow\; (k_x\hat{\boldsymbol{x}} + k_z\hat{\boldsymbol{z}})\times(E_{0x}\hat{\boldsymbol{x}} + E_{0z}\hat{\boldsymbol{z}}) = \mu_0\omega\boldsymbol{H}_0 \;\rightarrow\; H_{0y} = (\mu_0\omega)^{-1}(k_z E_{0x} - k_x E_{0z}). \tag{7}$$

The general expression for a p-polarized plane-wave's H-field is $\boldsymbol{H}(\boldsymbol{r},t) = H_{0y}\hat{\boldsymbol{y}}\, e^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$. The various H_{0y} are found from Eq.(7) with the aid of Eqs.(1), (2), (3), (5), (6) to be

$$H_{0y}^{(\mathrm{i})} = (\mu_0\omega)^{-1}[-(n_0\omega/c)\cos\theta - (n_0\omega/c)\sin\theta\tan\theta]E_{0x}^{(\mathrm{i})} = -\frac{n_0 E_{0x}^{(\mathrm{i})}}{Z_0\cos\theta}, \tag{8}$$

$$H_{0y}^{(\mathrm{r})} = (\mu_0\omega)^{-1}[(n_0\omega/c)\cos\theta + (n_0\omega/c)\sin\theta\tan\theta]E_{0x}^{(\mathrm{r})} = \frac{n_0 E_{0x}^{(\mathrm{r})}}{Z_0\cos\theta}, \tag{9}$$

$$H_{0y}^{(\mathrm{t})} = (\mu_0\omega)^{-1}\left[-(n_0\omega/c)\sqrt{(n_1/n_0)^2-\sin^2\theta} - \frac{(n_0\omega/c)\sin^2\theta}{\sqrt{(n_1/n_0)^2-\sin^2\theta}}\right]E_{0x}^{(\mathrm{t})} = -\frac{(n_1^2/n_0)E_{0x}^{(\mathrm{t})}}{Z_0\sqrt{(n_1/n_0)^2-\sin^2\theta}}. \tag{10}$$

b) At the interfacial xy-plane separating the incidence and transmittance media, the tangential component E_x of the E-field must be continuous, and so does the tangential component H_y of the H-field. Recalling that $\rho_p = E_{0x}^{(\mathrm{r})}/E_{0x}^{(\mathrm{i})}$ and $\tau_p = E_{0x}^{(\mathrm{t})}/E_{0x}^{(\mathrm{i})}$, we write

$$\text{Continuity of } \boldsymbol{E}_\parallel: \quad E_{0x}^{(\mathrm{i})} + E_{0x}^{(\mathrm{r})} = E_{0x}^{(\mathrm{t})} \;\rightarrow\; 1+\rho_p = \tau_p. \tag{11}$$

$$\text{Continuity of } \boldsymbol{H}_\parallel: \quad H_{0y}^{(\mathrm{i})} + H_{0y}^{(\mathrm{r})} = H_{0y}^{(\mathrm{t})} \;\rightarrow\; -\frac{n_0E_{0x}^{(\mathrm{i})}}{Z_0\cos\theta} + \frac{n_0E_{0x}^{(\mathrm{r})}}{Z_0\cos\theta} = -\frac{(n_1^2/n_0)E_{0x}^{(t)}}{Z_0\sqrt{(n_1/n_0)^2-\sin^2\theta}}$$

$$\rightarrow\; 1-\rho_p = \frac{(n_1/n_0)^2\cos\theta}{\sqrt{(n_1/n_0)^2-\sin^2\theta}}\tau_p. \tag{12}$$

c) Considering that $n^2(\omega) = \mu(\omega)\varepsilon(\omega) = 1+\chi_e(\omega)$, the material polarization within the transmittance medium should be written as

$$\boldsymbol{P}(\boldsymbol{r},t) = \varepsilon_0\chi_e(\omega)\boldsymbol{E}_0^{(\mathrm{t})}\exp[\mathrm{i}(\boldsymbol{k}^{(\mathrm{t})}\cdot\boldsymbol{r}-\omega t)] = \varepsilon_0[n_1^2(\omega)-1]\big(E_{0x}^{(\mathrm{t})}\hat{\boldsymbol{x}} + E_{0z}^{(\mathrm{t})}\hat{\boldsymbol{z}}\big)e^{\mathrm{i}(k_x x-\omega t)}e^{\mathrm{i}k_z^{(\mathrm{t})}z}. \tag{13}$$

The bound electric charge-density $\rho_{\text{bound}}^{(\mathrm{e})}(\boldsymbol{r},t) = -\boldsymbol{\nabla}\cdot\boldsymbol{P}(\boldsymbol{r},t) = -\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{P}_0 e^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$ vanishes everywhere within the transmittance medium, simply because $\boldsymbol{k}^{(\mathrm{t})}\cdot\boldsymbol{E}_0^{(\mathrm{t})} = 0$ (in accordance with Maxwell's 1st equation). As for the bound electric current-density, we will have

$$\boldsymbol{J}_{\text{bound}}^{(\mathrm{e})}(\boldsymbol{r},t) = \partial\boldsymbol{P}/\partial t = -\mathrm{i}\omega\varepsilon_0(n_1^2-1)(E_{0x}^{(\mathrm{t})}\hat{\boldsymbol{x}} + E_{0z}^{(\mathrm{t})}\hat{\boldsymbol{z}})e^{\mathrm{i}(k_x x-\omega t)}e^{\mathrm{i}k_z^{(\mathrm{t})}z}. \tag{14}$$

The above expression can be further streamlined by substituting for k_x and $k_z^{(\mathrm{t})}$ from Eq.(3), and for $E_{0z}^{(\mathrm{t})}$ from Eq.(6).

Digression. If the imaginary part of n_1 is taken to be positive, the imaginary part of $k_z^{(\mathrm{t})}$ will turn out to be negative. The integral of the bound current-density $\boldsymbol{J}_{\text{bound}}^{(\mathrm{e})}$ over the infinite depth of the transmittance medium can then be evaluated as follows:

$$\int_{z=-\infty}^{0}\boldsymbol{J}_{\text{bound}}^{(\mathrm{e})}(\boldsymbol{r},t)\mathrm{d}z = -\varepsilon_0(\omega/k_z^{(\mathrm{t})})(n_1^2-1)(E_{0x}^{(\mathrm{t})}\hat{\boldsymbol{x}} + E_{0z}^{(\mathrm{t})}\hat{\boldsymbol{z}})e^{\mathrm{i}(k_x x-\omega t)}\,e^{\mathrm{i}k_z^{(\mathrm{t})}z}\Big|_{z=-\infty}^{0}$$

$$= \frac{\varepsilon_0\omega(n_1^2-1)\tau_p}{(n_0\omega/c)\sqrt{(n_1/n_0)^2-\sin^2\theta}}\left[\hat{\boldsymbol{x}} + \frac{\sin\theta}{\sqrt{(n_1/n_0)^2-\sin^2\theta}}\hat{\boldsymbol{z}}\right]E_{0x}^{(\mathrm{i})}e^{\mathrm{i}(k_x x-\omega t)}. \tag{15}$$

Solving Eqs.(11) and (12) for ρ_p and τ_p, we arrive at the Fresnel reflection and transmission coefficients for p-polarized light, as follows:

$$\rho_p = \frac{\sqrt{(n_1/n_0)^2-\sin^2\theta} - (n_1/n_0)^2\cos\theta}{\sqrt{(n_1/n_0)^2-\sin^2\theta} + (n_1/n_0)^2\cos\theta}, \tag{16}$$

$$\tau_p = \frac{2\sqrt{(n_1/n_0)^2-\sin^2\theta}}{\sqrt{(n_1/n_0)^2-\sin^2\theta} + (n_1/n_0)^2\cos\theta}. \tag{17}$$

In the limit when $n_1 \to \infty$, we have $\sqrt{(n_1/n_0)^2 - \sin^2\theta} \to (n_1/n_0)$ and $\tau_p \to 2n_0/(n_1 \cos\theta)$. The integrated current density of Eq.(15) then approaches $2n_0 E_{0x}^{(\mathrm{i})} \hat{\boldsymbol{x}} e^{\mathrm{i}(k_x x - \omega t)}/(Z_0 \cos\theta)$. This can be interpreted as a surface-current-density $\boldsymbol{J}_s(x, y, z = 0^-, t)$ residing within the skin-depth of a highly conductive (or absorptive) transmittance medium. In the same limit, $\rho_p \to -1$, resulting in the tangential H-field at the xy-plane immediately above the interface to approach $H_{0y}^{(\mathrm{i})} + H_{0y}^{(\mathrm{r})} = -2n_0 E_{0x}^{(\mathrm{i})}/(Z_0 \cos\theta)$. Immediately below the interfacial xy-plane at $z = 0^-$, we have, in the limit,

$$H_{0y}^{(\mathrm{t})} = -\frac{(n_1^2/n_0 Z_0)\tau_p E_{0x}^{(\mathrm{i})}}{\sqrt{(n_1/n_0)^2 - \sin^2\theta}} \to -2n_0 E_{0x}^{(\mathrm{i})}/(Z_0 \cos\theta). \tag{18}$$

Consequently, the tangential H-field at the interface remains continuous even in the limit when $n_1 \to \infty$. However, in this limit, the fields inside the transmittance medium decay exponentially rapidly toward zero, indicating that slightly below the skin-depth (which has now shrunk to nothingness) the H-field vanishes. The discontinuity of the tangential H-field across the skin-depth is thus seen to be equal in magnitude and perpendicular in direction to the aforementioned surface-current-density $\boldsymbol{J}_s$.

It is also worthwhile to examine the boundary condition associated with the perpendicular D-field across the interfacial plane. Given that $\boldsymbol{D} = \varepsilon_0 \varepsilon(\omega)\boldsymbol{E} = \varepsilon_0 n^2 \boldsymbol{E}$, we will have

$$D_\perp(x, y, z = 0^+, t) = \varepsilon_0 n_0^2 (E_{0z}^{(\mathrm{i})} + E_{0z}^{(\mathrm{r})}) e^{\mathrm{i}(k_x x - \omega t)} = \varepsilon_0 n_0^2 \tan\theta\,(1 - \rho_p) E_{0x}^{(\mathrm{i})} e^{\mathrm{i}(k_x x - \omega t)}, \tag{19}$$

$$D_\perp(x, y, z = 0^-, t) = \varepsilon_0 n_1^2 E_{0z}^{(\mathrm{t})} e^{\mathrm{i}(k_x x - \omega t)} = \frac{\varepsilon_0 n_1^2 (\sin\theta)\tau_p}{\sqrt{(n_1/n_0)^2 - \sin^2\theta}} E_{0x}^{(\mathrm{i})} e^{\mathrm{i}(k_x x - \omega t)}. \tag{20}$$

Substituting for ρ_p from Eq.(16) into Eq.(19), and for τ_p from Eq.(17) into Eq.(20), it is now easy to confirm that indeed $D_\perp$ remains continuous across the interfacial xy-plane.

The bound electric charge-density was shown in part (c) to vanish everywhere inside the transmittance medium. A similar argument can be made for the absence of $\rho_{\mathrm{bound}}^{(\mathrm{e})}$ inside the incidence medium. However, the bound surface-charge-density is not zero at the interface between the two media. The material polarization of the incidence medium is given by

$$\boldsymbol{P}(\boldsymbol{r}, t) = \varepsilon_0 (n_0^2 - 1)\big[(E_{0x}^{(\mathrm{i})} \hat{\boldsymbol{x}} + E_{0z}^{(\mathrm{i})} \hat{\boldsymbol{z}}) e^{-\mathrm{i}(n_0\omega/c)\cos\theta z} + (E_{0x}^{(\mathrm{r})} \hat{\boldsymbol{x}} + E_{0z}^{(\mathrm{r})} \hat{\boldsymbol{z}}) e^{\mathrm{i}(n_0\omega/c)\cos\theta z}\big] e^{\mathrm{i}(k_x x - \omega t)} \mathrm{step}(z). \tag{21}$$

The bound electric charge-density $\rho_{\mathrm{bound}}^{(\mathrm{e})}(\boldsymbol{r}, t) = -\boldsymbol{\nabla} \cdot \boldsymbol{P}(\boldsymbol{r}, t)$ has an additional term arising from $\partial \mathrm{step}(z)/\partial z = \delta(z)$, which gives rise to a surface-charge-density at $z = 0^+$, as follows:

$$\sigma_s(x, y, z = 0^+, t) = -\varepsilon_0 (n_0^2 - 1)(E_{0z}^{(\mathrm{i})} + E_{0z}^{(\mathrm{r})}) e^{\mathrm{i}(k_x x - \omega t)}. \tag{22}$$

There is also a similar surface charge-density on the surface of the transmittance medium at $z = 0^-$, which is obtained from Eq.(13) as

$$\sigma_s(x, y, z = 0^-, t) = \varepsilon_0 (n_1^2 - 1) E_{0z}^{(\mathrm{t})} e^{\mathrm{i}(k_x x - \omega t)}. \tag{23}$$

The total surface charge-density is the sum of Eqs.(22) and (23). The terms corresponding to the continuity of $D_\perp$ add up to zero (see Eqs.(19) and (20)); what remains then is

$$\sigma_s(x, y, z = 0^+, t) + \sigma_s(x, y, z = 0^-, t) = \varepsilon_0 (E_{0z}^{(\mathrm{i})} + E_{0z}^{(\mathrm{r})} - E_{0z}^{(\mathrm{t})}) e^{\mathrm{i}(k_x x - \omega t)}. \tag{24}$$

This equation reveals that the discontinuity of the perpendicular E-field at the interfacial xy-plane equals the (bound) surface-charge-density σ_s divided by ε_0, precisely as expected from the boundary condition associated with Maxwell's 1st equation.
