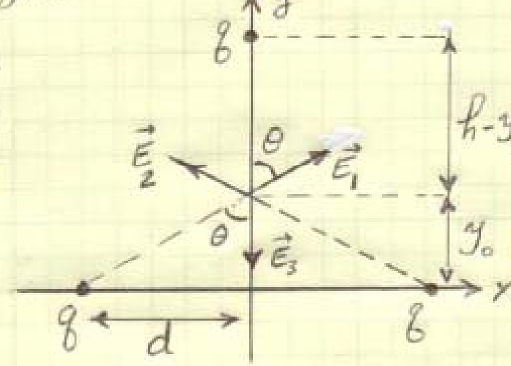
Problem 11)

a) Because of symmetry, the point of equilibrium is likely to be on the $y$-axis, therefore, $x_0 = 0$. We also expect $0 < y_0 < h$, otherwise the point will be outside the triangle, with all the charges pushing or pulling in the same direction. From the figure it is readily seen that, if the three bisectors of the equilateral triangle are drawn, they'll meet at a central point which is equi-distant from the three charges. Therefore, $|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3|$. Also, $\theta = 60°$, which means that the three field vectors have an angular separation of $120°$ from each other.



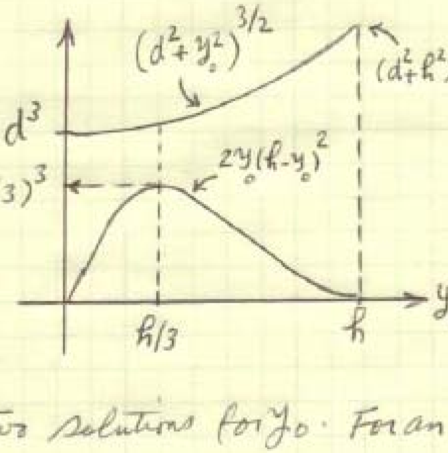

The net field, $\vec{E}_1 + \vec{E}_2 + \vec{E}_3$, is therefore equal to zero. We'll have:

$$\tan\theta = \frac{d}{y_0} \Rightarrow y_0 = \frac{d}{\tan 60°} = \frac{d}{\sqrt{3}} = \frac{\sqrt{3}d}{3} = h/3 \Rightarrow (x_0, y_0) = (0, \tfrac{h}{3})$$

* Digression: A more formal solution recognizes that $|\vec{E}_1| = |\vec{E}_2|$ because of symmetry. Equilibrium then demands that $|\vec{E}_3| = 2|\vec{E}_1|\cos\theta \Rightarrow$

$$\frac{q}{(h-y_0)^2} = 2\frac{q}{d^2+y_0^2}\cdot\frac{y_0}{\sqrt{d^2+y_0^2}} \Rightarrow (d^2+y_0^2)^{3/2} = 2y_0(h-y_0)^2$$

The two functions appearing in the above equation are plotted on the right. Clearly, there are no crossing points unless $(2h/3)^3 \geq (d^2 + \frac{h^2}{3^2})^{3/2} \Rightarrow h \geq \sqrt{3}d$.




In general, when $h > \sqrt{3}d$, there will be two solutions for $y_0$. For an equilateral triangle, however, $h=\sqrt{3}d$, and there is only one solution, namely, $y_0 = h/3$. (Note: When $h<\sqrt{3}d$, the above analysis does not imply that no equilibrium points exist; one must then re-examine the original assumption that $x_0=0$.)

b) There are no stable points of equilibrium in any electrostatic field. The reason is that, if a test charge placed at $(x_0, y_0)$ is moved away from $(x_0, y_0)$ by a small distance in an arbitrary direction, stability requires that the $\vec{E}$-field must always push it back toward $(x_0, y_0)$. This means that the integral of $\vec{E}$ over a closed surface surrounding $(x_0, y_0)$ must be either positive (i.e., all surrounding $\vec{E}$-fields pointing away from the equilibrium point) or negative (i.e., all surrounding $\vec{E}$-fields pointing toward the equilibrium point). In other words, $\vec{\nabla}\cdot\vec{E}\neq 0$ at the point of equilibrium. This is not possible, however, because, according to Gauss's law, $\vec{\nabla}\cdot\vec{E}=\rho/\epsilon_0$. Since there are no charges at the point $(x_0, y_0)$, we have $\rho=0$ and, consequently, $\vec{\nabla}\cdot\vec{E}=0$, contradicting the assumption of stability.