Problem 1) a) The correct expression of the Poynting vector is given in (ii), since it takes the *physical* E-field, which is the real part of the complex $\boldsymbol{E}(r,t)$, and cross-multiplies it into the *physical* H-field, which is the real part of the complex $\boldsymbol{H}(\boldsymbol{r},t)$. In general, the complex notation is used for mathematical convenience only; it does *not* represent the actual (i.e., physical) field.

The units of $\boldsymbol{E}$ (both its real and imaginary parts) are [volt/meter], while the units of $\boldsymbol{H}$ (both its real and imaginary parts) are [ampere/meter]. The cross-product of $\boldsymbol{E}$ and $\boldsymbol{H}$ (in both their real and complex representations) has units of [volt · ampere/m^2], which equals [watt/m^2] or [joule/(sec · m^2)].

b) (i) $\boldsymbol{S}(\boldsymbol{r},t) = \text{Real}\{\boldsymbol{E}(\boldsymbol{r},t) \times \boldsymbol{H}(\boldsymbol{r},t)\} = \text{Real}\{[\boldsymbol{E}'(\boldsymbol{r},t) + \text{i}\boldsymbol{E}''(\boldsymbol{r},t)] \times [\boldsymbol{H}'(\boldsymbol{r},t) + \text{i}\boldsymbol{H}''(\boldsymbol{r},t)]\}$

$$= \boldsymbol{E}'(\boldsymbol{r},t) \times \boldsymbol{H}'(\boldsymbol{r},t) - \boldsymbol{E}''(\boldsymbol{r},t) \times \boldsymbol{H}''(\boldsymbol{r},t).$$

(ii) $\boldsymbol{S}(\boldsymbol{r},t) = \text{Real}\{\boldsymbol{E}(\boldsymbol{r},t)\} \times \text{Real}\{\boldsymbol{H}(\boldsymbol{r},t)\} = \boldsymbol{E}'(\boldsymbol{r},t) \times \boldsymbol{H}'(\boldsymbol{r},t)$.

The extraneous term in (i) is $\boldsymbol{E}''(\boldsymbol{r},t) \times \boldsymbol{H}''(\boldsymbol{r},t)$, which makes the purported Poynting vector dependent on the imaginary parts of the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields, which are non-physical entities.

c) $\text{Real}\{\boldsymbol{E}(\boldsymbol{r},t)\} = \text{Real}\{[\boldsymbol{E}'(\boldsymbol{r}) + \text{i}\boldsymbol{E}''(\boldsymbol{r})][\cos(\omega t) - \text{i}\sin(\omega t)]\} = \boldsymbol{E}'(\boldsymbol{r})\cos(\omega t) + \boldsymbol{E}''(\boldsymbol{r})\sin(\omega t)$.

$\text{Real}\{\boldsymbol{H}(\boldsymbol{r},t)\} = \text{Real}\{[\boldsymbol{H}'(\boldsymbol{r}) + \text{i}\boldsymbol{H}''(\boldsymbol{r})][\cos(\omega t) - \text{i}\sin(\omega t)]\} = \boldsymbol{H}'(\boldsymbol{r})\cos(\omega t) + \boldsymbol{H}''(\boldsymbol{r})\sin(\omega t)$.

$$\boldsymbol{S}(\boldsymbol{r},t) = \text{Real}\{\boldsymbol{E}(\boldsymbol{r},t)\} \times \text{Real}\{\boldsymbol{H}(\boldsymbol{r},t)\} = \boldsymbol{E}'(\boldsymbol{r}) \times \boldsymbol{H}'(\boldsymbol{r})\cos^2(\omega t) + \boldsymbol{E}''(\boldsymbol{r}) \times \boldsymbol{H}''(\boldsymbol{r})\sin^2(\omega t)$$
$$+[\boldsymbol{E}'(\boldsymbol{r}) \times \boldsymbol{H}''(\boldsymbol{r}) + \boldsymbol{E}''(\boldsymbol{r}) \times \boldsymbol{H}'(\boldsymbol{r})]\sin(\omega t)\cos(\omega t)$$
$$= ½[\boldsymbol{E}'(\boldsymbol{r}) \times \boldsymbol{H}'(\boldsymbol{r}) + \boldsymbol{E}''(\boldsymbol{r}) \times \boldsymbol{H}''(\boldsymbol{r})] + ½[\boldsymbol{E}'(\boldsymbol{r}) \times \boldsymbol{H}'(\boldsymbol{r}) - \boldsymbol{E}''(\boldsymbol{r}) \times \boldsymbol{H}''(\boldsymbol{r})]\cos(2\omega t)$$
$$+½[\boldsymbol{E}'(\boldsymbol{r}) \times \boldsymbol{H}''(\boldsymbol{r}) + \boldsymbol{E}''(\boldsymbol{r}) \times \boldsymbol{H}'(\boldsymbol{r})]\sin(2\omega t).$$

d) Upon time-averaging, we find that $\int_{t_0}^{t_0+T}\cos(2\omega t)\,\text{d}t = 0$ and $\int_{t_0}^{t_0+T}\sin(2\omega t)\,\text{d}t = 0$. Therefore,

$$\langle\boldsymbol{S}(\boldsymbol{r},t)\rangle = ½[\boldsymbol{E}'(\boldsymbol{r}) \times \boldsymbol{H}'(\boldsymbol{r}) + \boldsymbol{E}''(\boldsymbol{r}) \times \boldsymbol{H}''(\boldsymbol{r})].$$

e) The real part of $\boldsymbol{E}(\boldsymbol{r}) \times \boldsymbol{H}^*(\boldsymbol{r})$ is readily computed, as follows:

$$\text{Real}\{\boldsymbol{E}(\boldsymbol{r}) \times \boldsymbol{H}^*(\boldsymbol{r})\} = \text{Real}\{[\boldsymbol{E}'(\boldsymbol{r}) + \text{i}\boldsymbol{E}''(\boldsymbol{r})] \times [\boldsymbol{H}'(\boldsymbol{r}) - \text{i}\boldsymbol{H}''(\boldsymbol{r})]\}$$
$$= \boldsymbol{E}'(\boldsymbol{r}) \times \boldsymbol{H}'(\boldsymbol{r}) + \boldsymbol{E}''(\boldsymbol{r}) \times \boldsymbol{H}''(\boldsymbol{r}).$$

A direct comparison with the time-averaged Poynting vector derived in part (d) now shows that

$$\langle\boldsymbol{S}(\boldsymbol{r},t)\rangle = ½\text{Real}\{\boldsymbol{E}(\boldsymbol{r}) \times \boldsymbol{H}^*(\boldsymbol{r})\}.$$

Problem 2) a) Within the incidence medium, we have

$$\boldsymbol{k}^{(\text{i})} = -(\omega/c)\hat{\boldsymbol{z}}, \qquad \boldsymbol{E}_0^{(\text{i})} = E_{0x}^{(\text{i})}\hat{\boldsymbol{x}}, \qquad \boldsymbol{H}_0^{(\text{i})} = \boldsymbol{k}^{(\text{i})} \times \boldsymbol{E}_0^{(\text{i})}/(\mu_0\omega) = -(E_{0x}^{(\text{i})}/Z_0)\hat{\boldsymbol{y}} = H_{0y}^{(\text{i})}\hat{\boldsymbol{y}}.$$

Therefore,

$$\boldsymbol{E}^{(\text{i})}(\boldsymbol{r},t) = \boldsymbol{E}_0^{(\text{i})}e^{\text{i}(\boldsymbol{k}^{(\text{i})}\cdot\boldsymbol{r} - \omega t)} = E_{0x}^{(\text{i})}\hat{\boldsymbol{x}}e^{-\text{i}(\omega/c)(z+ct)}. \qquad (1)$$

$$\boldsymbol{H}^{(\text{i})}(\boldsymbol{r},t) = \boldsymbol{H}_0^{(\text{i})}e^{\text{i}(\boldsymbol{k}^{(\text{i})}\cdot\boldsymbol{r} - \omega t)} = -(E_{0x}^{(\text{i})}/Z_0)\hat{\boldsymbol{y}}e^{-\text{i}(\omega/c)(z+ct)}. \qquad (2)$$

For the reflected plane-wave, the formulas are similar to those of the incident wave, except that $\boldsymbol{k}^{(\mathrm{r})} = (\omega/c)\hat{\boldsymbol{z}}$, $\boldsymbol{E}_0^{(\mathrm{r})} = \rho E_{0x}^{(\mathrm{i})}\hat{\boldsymbol{x}}$, and $\boldsymbol{H}_0^{(\mathrm{r})} = \boldsymbol{k}^{(\mathrm{r})} \times \boldsymbol{E}_0^{(\mathrm{r})}/(\mu_0\omega) = (\rho E_{0x}^{(\mathrm{i})}/Z_0)\hat{\boldsymbol{y}}$. Therefore,

$$\boldsymbol{E}^{(\mathrm{r})}(\boldsymbol{r},t) = \boldsymbol{E}_0^{(\mathrm{r})} e^{\mathrm{i}(\boldsymbol{k}^{(\mathrm{r})}\cdot\boldsymbol{r} - \omega t)} = \rho E_{0x}^{(\mathrm{i})}\hat{\boldsymbol{x}} e^{\mathrm{i}(\omega/c)(z-ct)}. \tag{3}$$

$$\boldsymbol{H}^{(\mathrm{r})}(\boldsymbol{r},t) = \boldsymbol{H}_0^{(\mathrm{r})} e^{\mathrm{i}(\boldsymbol{k}^{(\mathrm{r})}\cdot\boldsymbol{r} - \omega t)} = (\rho\, E_{0x}^{(\mathrm{i})}/Z_0)\hat{\boldsymbol{y}} e^{\mathrm{i}(\omega/c)(z-ct)}. \tag{4}$$

For the transmitted beam, the generalized Snell's law guarantees that $k_x^{(\mathrm{t})} = k_y^{(\mathrm{t})} = 0$; the dispersion relation then yields $\boldsymbol{k}^{(\mathrm{t})} = k_z^{(\mathrm{t})}\hat{\boldsymbol{z}} = [(\omega/c)^2 n^2(\omega) - (k_x^{(\mathrm{t})})^2 - (k_y^{(\mathrm{t})})^2]^{½}\hat{\boldsymbol{z}} = \pm(\omega/c)n(\omega)\hat{\boldsymbol{z}}$. The correct sign for $k_z^{(\mathrm{t})}$ is minus, since the fields must decay inside the metal as $z \to -\infty$. We also have $\boldsymbol{E}_0^{(\mathrm{t})} = \tau E_{0x}^{(\mathrm{i})}\hat{\boldsymbol{x}}$, and $\boldsymbol{H}_0^{(\mathrm{t})} = \boldsymbol{k}^{(\mathrm{t})} \times \boldsymbol{E}_0^{(\mathrm{t})}/[\mu_0\mu(\omega)\omega] = -[\tau n(\omega) E_{0x}^{(\mathrm{i})}/Z_0]\hat{\boldsymbol{y}}$. Therefore,

$$\boldsymbol{E}^{(\mathrm{t})}(\boldsymbol{r},t) = \tau E_{0x}^{(\mathrm{i})}\hat{\boldsymbol{x}} e^{\mathrm{i}(\boldsymbol{k}^{(\mathrm{t})}\cdot\boldsymbol{r} - \omega t)} = \tau E_{0x}^{(\mathrm{i})}\hat{\boldsymbol{x}} e^{-\mathrm{i}(\omega/c)(nz+ct)}. \tag{5}$$

$$\boldsymbol{H}^{(\mathrm{t})}(\boldsymbol{r},t) = \boldsymbol{H}_0^{(\mathrm{t})} e^{\mathrm{i}(\boldsymbol{k}^{(\mathrm{t})}\cdot\boldsymbol{r} - \omega t)} = -[\tau n(\omega) E_{0x}^{(\mathrm{i})}/Z_0]\hat{\boldsymbol{y}} e^{-\mathrm{i}(\omega/c)(nz+ct)}. \tag{6}$$

b) Inside the metallic medium, we have

$$\boldsymbol{P}(\boldsymbol{r},t) = \varepsilon_0\chi_e(\omega)\boldsymbol{E}^{(\mathrm{t})}(\boldsymbol{r},t) \to \boldsymbol{J}_{\text{bound}}^{(e)}(\boldsymbol{r},t) = \partial\boldsymbol{P}(\boldsymbol{r},t)/\partial t = -\mathrm{i}\omega\varepsilon_0\chi_e(\omega)\tau E_{0x}^{(\mathrm{i})} e^{-\mathrm{i}(\omega/c)(nz+ct)}\hat{\boldsymbol{x}}. \tag{7}$$

Integrating the bound current density through the thickness of the metallic medium, we find

$$\begin{aligned}\boldsymbol{J}_{\mathrm{s}}^{(e)}(t) &= \int_{-\infty}^{0} \boldsymbol{J}_{\text{bound}}^{(e)}(\boldsymbol{r},t)\mathrm{d}z = -\mathrm{i}\omega\varepsilon_0\chi_e(\omega)\tau E_{0x}^{(\mathrm{i})} e^{-\mathrm{i}\omega t}\Big[\int_{-\infty}^{0} e^{-\mathrm{i}(\omega/c)n(\omega)z}\mathrm{d}z\Big]\hat{\boldsymbol{x}} \\ &= \frac{-\mathrm{i}\omega\varepsilon_0}{-\mathrm{i}(\omega/c)n(\omega)}[\varepsilon(\omega) - 1]\frac{2}{1+n(\omega)} E_{0x}^{(\mathrm{i})} e^{-\mathrm{i}\omega t}\hat{\boldsymbol{x}} \\ &= 2\varepsilon_0 c\left\{\frac{n^2(\omega) - 1}{n(\omega)[1+n(\omega)]}\right\} E_{0x}^{(\mathrm{i})} e^{-\mathrm{i}\omega t}\hat{\boldsymbol{x}} = 2\left[\frac{n(\omega)-1}{n(\omega)}\right]\left(E_{0x}^{(\mathrm{i})}/Z_0\right) e^{-\mathrm{i}\omega t}\hat{\boldsymbol{x}}. \end{aligned} \tag{8}$$

c) In the limit of $n''(\omega) \to \infty$, we will have $[n(\omega) - 1]/n(\omega) \to 1$, while the penetration depth inside the metallic medium approaches zero. Thus, the surface current-density of Eq.(8) becomes

$$lim_{n''(\omega)\to\infty}\boldsymbol{J}_{\mathrm{s}}^{(e)}(t) = 2(E_{0x}^{(\mathrm{i})}/Z_0)e^{-\mathrm{i}\omega t}\hat{\boldsymbol{x}}. \tag{9}$$

In this limit, the Fresnel reflection coefficient ρ approaches -1, and $\boldsymbol{H}_0^{(\mathrm{r})} \to -(E_{0x}^{(\mathrm{i})}/Z_0)\hat{\boldsymbol{y}}$. The total H-field immediately above the surface will then be $\boldsymbol{H}^{(\mathrm{i})} + \boldsymbol{H}^{(\mathrm{r})} = -2(E_{0x}^{(\mathrm{i})}/Z_0)e^{-\mathrm{i}\omega t}\hat{\boldsymbol{y}}$. Inside the metallic medium, the transmitted H-field rapidly drops to zero as $n''(\omega) \to \infty$, so that the discontinuity of the H-field (immediately above and slightly below the surface) now equals $-2(E_{0x}^{(\mathrm{i})}/Z_0)e^{-\mathrm{i}\omega t}\hat{\boldsymbol{y}}$. This discontinuity is equal in magnitude and perpendicular in direction to the surface-current-density $\boldsymbol{J}_{\mathrm{s}}^{(e)}(t)$ of Eq.(9), consistent with Maxwell's 2nd boundary condition.

Problem 3) a) The incident plane-wave is homogeneous and propagates along the x-axis; therefore, $k_y^{(\mathrm{i})} = k_z^{(\mathrm{i})} = 0$. The dispersion relation now yields $\boldsymbol{k}^{(\mathrm{i})} \cdot \boldsymbol{k}^{(\mathrm{i})} = (k_x^{(\mathrm{i})})^2 = (\omega/c)^2\mu_a\varepsilon_a = (\omega n_a/c)^2$. Consequently, $\boldsymbol{k}^{(\mathrm{i})} = k_x^{(\mathrm{i})}\hat{\boldsymbol{x}} = (n_a\omega/c)\hat{\boldsymbol{x}}$.

b) $\boldsymbol{k}^{(\mathrm{i})} \cdot \boldsymbol{E}^{(\mathrm{i})} = 0 \quad\to\quad k_x^{(\mathrm{i})}E_{0x}^{(\mathrm{i})} + \cancelto{0}{k_y^{(\mathrm{i})}} E_{0y}^{(\mathrm{i})} + \cancelto{0}{k_z^{(\mathrm{i})}} E_{0z}^{(\mathrm{i})} = 0 \quad\to\quad E_{0x}^{(\mathrm{i})} = 0.$

c) $\boldsymbol{k}^{(\mathrm{i})} \times \boldsymbol{E}_0^{(\mathrm{i})} = \omega\mu_0\mu_a\boldsymbol{H}_0^{(\mathrm{i})} \quad\to\quad \boldsymbol{H}_0^{(\mathrm{i})} = \dfrac{(n_a\omega/c)\hat{\boldsymbol{x}} \times (E_{0y}^{(\mathrm{i})}\hat{\boldsymbol{y}} + E_{0z}^{(\mathrm{i})}\hat{\boldsymbol{z}})}{\mu_0\mu_a\omega} = \sqrt{\varepsilon_0\varepsilon_a/\mu_0\mu_a}\left(E_{0y}^{(\mathrm{i})}\hat{\boldsymbol{z}} - E_{0z}^{(\mathrm{i})}\hat{\boldsymbol{y}}\right).$

d) $k_x^{(\mathrm{t})} = k_x^{(\mathrm{i})} = n_a\omega/c\,; \qquad k_y^{(\mathrm{t})} = k_y^{(\mathrm{i})} = 0.$

e) $\boldsymbol{k}^{(\mathrm{t})} \cdot \boldsymbol{k}^{(\mathrm{t})} = (k_x^{(\mathrm{t})})^2 + (\cancelto{0}{k_y^{(\mathrm{t})}})^2 + (k_z^{(\mathrm{t})})^2 = (\omega/c)^2 \mu_b \varepsilon_b \quad \rightarrow \quad k_z^{(\mathrm{t})} = \pm[(n_b \omega/c)^2 - (k_x^{(\mathrm{t})})^2]^{½}$

$$\rightarrow \quad k_z^{(\mathrm{t})} = \pm[(n_b \omega/c)^2 - (n_a \omega/c)^2]^{½} = \pm(\omega/c)\sqrt{n_b^2 - n_a^2} = \pm(\omega/c)\sqrt{\mu_b \varepsilon_b - \mu_a \varepsilon_a}.$$

The $\pm$ sign in the above expression of $k_z^{(\mathrm{t})}$ indicates that, in principle, both signs are viable. However, since the transmittance medium is taken to be semi-infinite, the resident plane-wave's exponential factor $\exp[\mathrm{i}(\boldsymbol{k}^{(\mathrm{t})} \cdot \boldsymbol{r} - \omega t)] = \exp(-k_z^{(t)} z) \exp[\mathrm{i}(k_x^{(\mathrm{t})} x + k_y^{(\mathrm{t})} y - \omega t)]$ must decay away from the interface. The sign of the (generally complex) square root in the above expression of $k_z^{(\mathrm{t})}$ must be chosen such that $\exp(-k_z^{(t)} z) \rightarrow 0$ as $z \rightarrow -\infty$. We thus find

$$\boldsymbol{k}^{(\mathrm{t})} = (\omega/c)(n_a \hat{\boldsymbol{x}} + \sqrt{\mu_b \varepsilon_b - \mu_a \varepsilon_a} \hat{\boldsymbol{z}}). \tag{1}$$

Note that $k_z^{(t)}$ of the transmitted beam is *not* directly related to $k_z^{(\mathrm{i})}$ of the incident beam, the latter being given by $k_z^{(\mathrm{i})} = \pm[(n_a \omega/c)^2 - (k_x^{(\mathrm{i})})^2]^{½} = 0$.

f) Continuity of $\boldsymbol{E}_\parallel$ at the interfacial xy-plane yields $E_{0x}^{(\mathrm{t})} = E_{0x}^{(\mathrm{i})} = 0$ and $E_{0y}^{(\mathrm{t})} = E_{0y}^{(\mathrm{i})}$.

g) $D_{0z}^{(\mathrm{t})} = D_{0z}^{(\mathrm{i})} \quad \rightarrow \quad \varepsilon_0 \varepsilon_b E_{0z}^{(\mathrm{t})} = \varepsilon_0 \varepsilon_a E_{0z}^{(\mathrm{i})} \quad \rightarrow \quad E_{0z}^{(\mathrm{t})} = (\varepsilon_a/\varepsilon_b) E_{0z}^{(\mathrm{i})}$.

h) In the absence of surface currents, the continuity of $\boldsymbol{H}_\parallel$ at the interfacial xy-plane yields $H_{0x}^{(\mathrm{t})} = H_{0x}^{(\mathrm{i})} = 0$ and $H_{0y}^{(\mathrm{t})} = H_{0y}^{(\mathrm{i})} = -\sqrt{\varepsilon_0 \varepsilon_a/\mu_0 \mu_a}\, E_{0z}^{(\mathrm{i})} = -\sqrt{\varepsilon_a/\mu_a}\, E_{0z}^{(\mathrm{i})}/Z_0$.

i) Continuity of $\boldsymbol{B}_\perp$ at the interfacial xy-plane yields $B_{0z}^{(\mathrm{t})} = B_{0z}^{(\mathrm{i})}$; therefore,

$$\mu_0 \mu_b H_{0z}^{(\mathrm{t})} = \mu_0 \mu_a H_{0z}^{(\mathrm{i})} \rightarrow H_{0z}^{(\mathrm{t})} = (\mu_a/\mu_b)\sqrt{\varepsilon_0 \varepsilon_a/\mu_0 \mu_a}\, E_{0y}^{(\mathrm{i})} = \sqrt{\mu_a \varepsilon_a/\mu_b^2}\, E_{0y}^{(\mathrm{i})}/Z_0 = n_a E_{0y}^{(\mathrm{i})}/(Z_0 \mu_b).$$

All in all, the transmitted $\boldsymbol{E}$ and $\boldsymbol{H}$ field-amplitudes are seen to be

$$\boldsymbol{E}_0^{(\mathrm{t})} = E_{0y}^{(\mathrm{i})} \hat{\boldsymbol{y}} + (\varepsilon_a/\varepsilon_b) E_{0z}^{(\mathrm{i})} \hat{\boldsymbol{z}}. \tag{2}$$

$$Z_0 \boldsymbol{H}_0^{(\mathrm{t})} = -\sqrt{\varepsilon_a/\mu_a}\, E_{0z}^{(\mathrm{i})} \hat{\boldsymbol{y}} + (n_a/\mu_b) E_{0y}^{(\mathrm{i})}\, \hat{\boldsymbol{z}}. \tag{3}$$

In what follows, we verify that the transmitted plane-wave violates at least one of Maxwell's equations.

i) $\boldsymbol{k}^{(\mathrm{t})} \cdot \boldsymbol{D}_0^{(\mathrm{t})} = (\omega/c)\big(n_a \hat{\boldsymbol{x}} + \sqrt{\mu_b \varepsilon_b - \mu_a \varepsilon_a} \hat{\boldsymbol{z}}\big) \cdot \varepsilon_0 \varepsilon_b \big[E_{0y}^{(\mathrm{i})} \hat{\boldsymbol{y}} + (\varepsilon_a/\varepsilon_b) E_{0z}^{(\mathrm{i})} \hat{\boldsymbol{z}}\big]$

$$= \varepsilon_0 \varepsilon_a \sqrt{\mu_b \varepsilon_b - \mu_a \varepsilon_a}(\omega/c) E_{0z}^{(\mathrm{i})} \neq 0, \quad \text{(violation occurs for } p\text{-polarized light).}$$

ii) $\boldsymbol{k}^{(\mathrm{t})} \times \boldsymbol{H}_0^{(\mathrm{t})} = (\omega/c)\big(n_a \hat{\boldsymbol{x}} + \sqrt{\mu_b \varepsilon_b - \mu_a \varepsilon_a} \hat{\boldsymbol{z}}\big) \times \big[-\sqrt{\varepsilon_a/\mu_a}\, E_{0z}^{(\mathrm{i})} \hat{\boldsymbol{y}} + (n_a/\mu_b) E_{0y}^{(\mathrm{i})}\, \hat{\boldsymbol{z}}\big]/Z_0$

$$= \varepsilon_0 \varepsilon_a \omega \big[\sqrt{(\mu_b \varepsilon_b/\mu_a \varepsilon_a) - 1}\, E_{0z}^{(\mathrm{i})}\, \hat{\boldsymbol{x}} - (\mu_a/\mu_b) E_{0y}^{(\mathrm{i})} \hat{\boldsymbol{y}} - E_{0z}^{(\mathrm{i})} \hat{\boldsymbol{z}}\big].$$

The above expression differs from $-\omega \boldsymbol{D}_0^{(\mathrm{t})} = -\omega \varepsilon_0 \varepsilon_b \boldsymbol{E}_0^{(\mathrm{t})}$ in both its x and y components. The violation of this 2nd of Maxwell's equations occurs for p- as well as s-polarized light.

iii) $\boldsymbol{k}^{(\mathrm{t})} \times \boldsymbol{E}_0^{(\mathrm{t})} = (\omega/c)\big(n_a \hat{\boldsymbol{x}} + \sqrt{\mu_b \varepsilon_b - \mu_a \varepsilon_a} \hat{\boldsymbol{z}}\big) \times \big[E_{0y}^{(\mathrm{i})} \hat{\boldsymbol{y}} + (\varepsilon_a/\varepsilon_b) E_{0z}^{(\mathrm{i})} \hat{\boldsymbol{z}}\big]$

$$= -(\omega/c)\big[\sqrt{\mu_b \varepsilon_b - \mu_a \varepsilon_a} E_{0y}^{(\mathrm{i})}\, \hat{\boldsymbol{x}} + n_a (\varepsilon_a/\varepsilon_b) E_{0z}^{(\mathrm{i})} \hat{\boldsymbol{y}} - n_a E_{0y}^{(\mathrm{i})} \hat{\boldsymbol{z}}\big].$$

The above expression differs from $\omega \boldsymbol{B}_0^{(\mathrm{t})} = \omega \mu_0 \mu_b \boldsymbol{H}_0^{(\mathrm{t})}$ in both its x and y components. The violation of this 3rd of Maxwell's equations occurs for p- as well as s-polarized light.

iv) $\boldsymbol{k}^{(\mathrm{t})} \cdot \boldsymbol{B}_0^{(\mathrm{t})} = (\omega/c)\big(n_a \hat{\boldsymbol{x}} + \sqrt{\mu_b \varepsilon_b - \mu_a \varepsilon_a} \hat{\boldsymbol{z}}\big) \cdot \mu_0 \mu_b \big[-\sqrt{\varepsilon_a/\mu_a}\, E_{0z}^{(\mathrm{i})} \hat{\boldsymbol{y}} + (n_a/\mu_b) E_{0y}^{(\mathrm{i})} \hat{\boldsymbol{z}}\big]/Z_0$

$$= (n_a\omega/c^2)\sqrt{\mu_b\varepsilon_b - \mu_a\varepsilon_a}E_{0y}^{(\mathrm{i})} \neq 0, \qquad \text{(violation occurs for } s\text{-polarized light).}$$

Thus, under no circumstances will it be possible to have an incident plane-wave at grazing incidence *without* the corresponding reflected wave. The Fresnel reflection and transmission coefficients confirm this conclusion since, at grazing incidence, $\rho_p = \rho_s = -1$ and $\tau_p = \tau_s = 0$. The fact that the reflection coefficients are equal to -1 indicates that, at grazing incidence, the incident and reflected beams cancel each other out.
