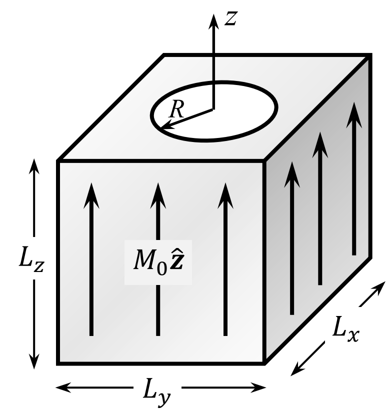
Opti 501 **2nd Midterm Exam** (11/8/2016) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

Problem 1) A solid rectangular block of dimensions $L_x \times L_y \times L_z$ with a central cylindrical hole of radius R, has a uniform, time-independent magnetization $M_0\hat{\boldsymbol{z}}$, as shown.

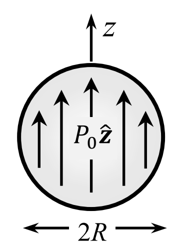


2 pts a) Use the special functions Rect(·) and Circ(·) to describe the magnetization distribution $\boldsymbol{M}(\boldsymbol{r},t)$ as a function of space and time.

3 pts b) Determine the bound magnetic charge-density distribution $\rho^{(m)}_{\text{bound}}(\boldsymbol{r},t)$.

3 pts c) Determine the bound electric current-density distribution $\boldsymbol{J}^{(e)}_{\text{bound}}(\boldsymbol{r},t)$.

Problem 2) A uniformly-polarized spherical particle of radius R has the polarization distribution $\boldsymbol{P}(\boldsymbol{r},t) = P_0\hat{\boldsymbol{z}}\,\text{Sphere}(r/R)$. The scalar potential for this particle is known to be



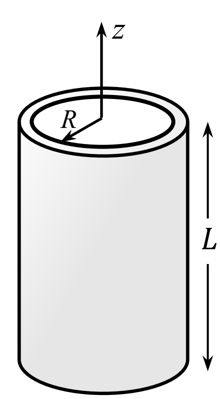
$$\psi(\boldsymbol{r},t) = \begin{cases} \dfrac{P_0 r\cos\theta}{3\varepsilon_0}; & r \le R, \\ \dfrac{P_0 R^3\cos\theta}{3\varepsilon_0 r^2}; & r \ge R. \end{cases}$$

3 pts a) Find the E-field distribution inside as well as outside the sphere.

3 pts b) Determine the bound electric charge-density distribution $\rho^{(e)}_{\text{bound}}$ on the surface of the sphere.

2 pts c) Show that, at the sphere's surface, the relevant Maxwell boundary conditions are satisfied.

Problem 3) A perfectly electrically conducting can is a hollow right-circular cylinder of radius R and length L, as shown. Inside the can, there exists a trapped electromagnetic wave, whose $\boldsymbol{E}$ and $\boldsymbol{H}$ fields are given by



$$\boldsymbol{E}(\boldsymbol{r},t) = E_0\,J_0(\omega_0 r_\parallel/c)\sin(\omega_0 t)\,\hat{\boldsymbol{z}},$$

$$\boldsymbol{H}(\boldsymbol{r},t) = (E_0/Z_0)\,J_1(\omega_0 r_\parallel/c)\cos(\omega_0 t)\,\hat{\boldsymbol{\phi}}.$$

Here $J_0(\cdot)$ and $J_1(\cdot)$ are Bessel functions of the first kind, orders 0 and 1, respectively, and the position coordinates are denoted by $(r_\parallel, \phi, z)$, where $r_\parallel$ is the length of the radial vector $\boldsymbol{r}_\parallel = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}}$. The speed of light in vacuum is $c = 1/\sqrt{\mu_0\varepsilon_0}$, and the impedance of free space is $Z_0 = \sqrt{\mu_0/\varepsilon_0}$.

3 pts a) Under what circumstances will the trapped wave exist within the enclosed cylindrical cavity?

2 pts b) Find the surface electric charge-density on the interior facets of the cylinder. (Note that, in addition to the cylindrical wall, the cavity is capped with flat plates at the top and bottom.)

3 pts c) Find the surface electric current-density on all the interior surfaces of the cylindrical cavity.

1 pt d) Confirm that the charge-current continuity equation is satisfied everywhere on the interior surfaces of the enclosed cavity.

Hint: The Bessel function identity $J_1(x) + xJ_1'(x) = xJ_0(x)$ [Chapter 3, Eq.(41)] may be helpful.
