Opti 501 **2^{nd} Midterm Exam** (10/30/2014) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

Problem 1) Use special functions such as $\delta(\cdot)$, Rect$(\cdot)$, and Circ$(\cdot)$ to describe the following physical systems.

1 Pt a) Thin disk of radius R, located in the xy-plane and centered at the origin of coordinates, having uniform charge distribution with surface-charge-density σ_{s0} (units $= coulomb/m^2$).

1 Pt b) Rectangular parallelepiped (or cuboid) of length L_x, width L_y, and height L_z, centered at the origin of coordinates and uniformly filled with electric dipoles, having a constant polarization P_0 along the x-axis.

2 Pts c) Cylinder of radius R and height h, centered at the origin of coordinates, with its cylinder axis in the z-direction, uniformly filled with magnetic dipoles, having a magnetization that oscillates as a function of time with frequency ω_0 and amplitude M_0 (along the z-axis).

Problem 2) A plane electromagnetic wave traveling in free space along the y-axis is reflected from a perfect electrically-conducting (PEC) mirror, as shown in the figure. In the region $y \le 0$, the incident and reflected potentials in the Lorenz gauge are given by

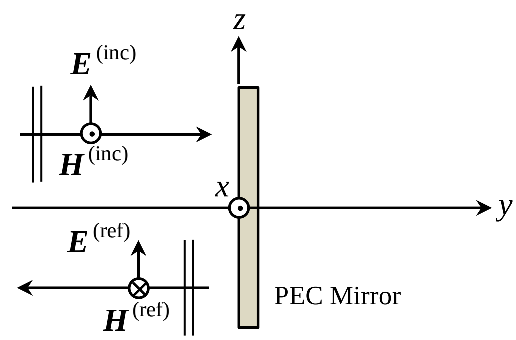
$$\psi^{(\text{inc})}(\boldsymbol{r},t) = 0,$$

$$\boldsymbol{A}^{(\text{inc})}(\boldsymbol{r},t) = A_0\hat{\boldsymbol{z}}\sin(k_0 y - \omega_0 t),$$

$$\psi^{(\text{ref})}(\boldsymbol{r},t) = 0,$$

$$\boldsymbol{A}^{(\text{ref})}(\boldsymbol{r},t) = A_0\hat{\boldsymbol{z}}\sin(k_0 y + \omega_0 t),$$

where A_0, k_0, and ω_0 are real-valued constants, with $k_0 = \omega_0/c$.



3 Pts a) Determine the electric field $\boldsymbol{E}(\boldsymbol{r},t)$ and the magnetic field $\boldsymbol{H}(\boldsymbol{r},t)$ in the region $y \le 0$.

2 Pts b) Use Maxwell's boundary conditions at the front facet of the mirror (i.e., in the xz-plane at $y = 0$) to determine the densities of surface charge distribution $\sigma_s(x,z,t)$ and surface current distribution $\boldsymbol{J}_s(x,z,t)$ at the mirror surface.

3 Pts c) The sheet of current induced at the front facet of the mirror must radiate both forward- and backward-propagating plane-waves in accordance with Example 10, Chapter 4. Considering that no light reaches inside the shadow of the PEC mirror (i.e., the region $y \ge 0$ remains dark), explain what happens to the plane-wave that is radiated into $y \ge 0$ by the oscillating surface current $\boldsymbol{J}_s(x,z,t)$.

Problem 3) A plane-wave in free space is specified in the Lorenz gauge by its scalar potential $\psi(\boldsymbol{r},t)=\psi_0\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]$ and its vector potential $\boldsymbol{A}(\boldsymbol{r},t)=\boldsymbol{A}_0\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]$.

2 Pts a) Specify the relation among ψ_0, $\boldsymbol{A}_0$, $\boldsymbol{k}$, and ω, considering that the potentials satisfy the Lorenz gauge.

2 Pts b) In terms of ψ_0, $\boldsymbol{A}_0$, $\boldsymbol{k}$, and ω, specify the electric field $\boldsymbol{E}(\boldsymbol{r},t)$ and the magnetic field $\boldsymbol{B}(\boldsymbol{r},t)$ associated with the above plane-wave.

3 Pts c) Considering that, in free space, $\rho_{\text{free}}(\boldsymbol{r},t)=0$, $\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t)=0$, $\boldsymbol{P}(\boldsymbol{r},t)=0$, and $\boldsymbol{M}(\boldsymbol{r},t)=0$, write Maxwell's equations for the above plane-wave, then explore the conditions under which all four equations are satisfied. (**Hint**: You will find that k must be related to ω.)

Problem 4) Let the electric charge-density distribution throughout the entire space and for all times be given by $\rho(\boldsymbol{r},t)=\rho_0\cos(k_0x)\cos(\omega_0 t)$, where ρ_0 is a real-valued constant, while k_0 and ω_0 are real, positive constants.

2 Pts a) Use the charge-current continuity equation $\boldsymbol{\nabla}\cdot\boldsymbol{J}+\partial\rho/\partial t=0$, to determine the electric current-density distribution $\boldsymbol{J}(\boldsymbol{r},t)$. (**Hint**: Constants of integration may be ignored.)

2 Pts b) Determine the scalar and vector potentials $\psi(\boldsymbol{r},t)$ and $\boldsymbol{A}(\boldsymbol{r},t)$ in the Lorenz gauge, assuming that $k_0\neq(\omega_0/c)$, where c is the speed of light in vacuum.

2 Pts c) Find the electric field $\boldsymbol{E}(\boldsymbol{r},t)$ and the magnetic field $\boldsymbol{B}(\boldsymbol{r},t)$ throughout the entire space-time.

Hint: You may find the identities $\cos x=\frac{1}{2}[\exp(\mathrm{i}x)+\exp(-\mathrm{i}x)]$ and $\sin x=\frac{1}{2\mathrm{i}}[\exp(\mathrm{i}x)-\exp(-\mathrm{i}x)]$ useful.
