Opti 501 **2^{nd} Midterm Exam** (11/13/2012) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together. Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

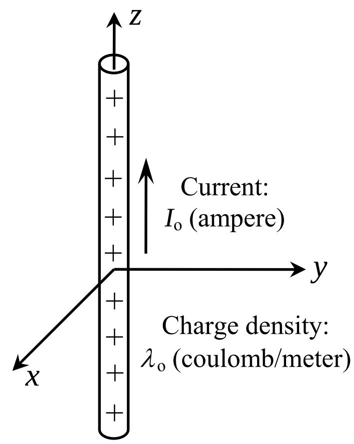
Problem 1) A static magnetic point-dipole $\boldsymbol{M}(\boldsymbol{r}) = m_0\delta(x)\delta(y)\delta(z)\hat{\boldsymbol{z}}$ sits at the origin of a Cartesian coordinate system.

2 Pts a) Find the bound current-density $\boldsymbol{J}^{(e)}_{\text{bound}}$ associated with the magnetic point-dipole.

3 Pts b) Using the formula $\boldsymbol{A}(\boldsymbol{r}) = (\mu_o/4\pi)\int_{-\infty}^{\infty}[\boldsymbol{J}(\boldsymbol{r}')/|\boldsymbol{r}-\boldsymbol{r}'|]\mathrm{d}\boldsymbol{r}'$, find the vector potential in the space surrounding the point-dipole.

3 Pts c) Determine the magnetic field $\boldsymbol{B}(\boldsymbol{r})$ of the point-dipole.

Problem 2) An infinitely-long, thin wire has a constant, uniform charge density λ_o [coulomb/m], and carries a constant, uniform current I_o [ampere] along the z-axis, as shown.



3 Pts a) Use the formula $\psi(\boldsymbol{r}) = (4\pi\varepsilon_o)^{-1}\int_{-\infty}^{\infty}[\rho(\boldsymbol{r}')/|\boldsymbol{r}-\boldsymbol{r}'|]\mathrm{d}\boldsymbol{r}'$ to calculate the scalar potential of the wire in its surrounding space.

Hint: Assume at first that the wire extends from $-z_0$ to z_0, with z_0 being large but finite. You will need the following integral:

$$\int_0^{z_0}\mathrm{d}z'/\sqrt{\rho^2+z'^2} = \ln(z'+\sqrt{\rho^2+z'^2})\Big|_0^{z_0} = \ln(z_0+\sqrt{\rho^2+z_0^2}) - \ln\rho.$$

2 Pts b) To justify the neglect of $\ln(z_0+\sqrt{\rho^2+z_0^2})$ in the final expression of $\psi(\boldsymbol{r})$, one will have to argue that the neglected term, although infinitely large, does *not* contribute to the E-field. Considering that $\boldsymbol{E}(\boldsymbol{r}) = -\boldsymbol{\nabla}\psi(\boldsymbol{r})$, find the contribution of the neglected term to $\boldsymbol{E}(\boldsymbol{r})$ and show that, in the limit when $z_0\to\infty$, this neglect is justified.

2 Pts c) Use the formula $\boldsymbol{A}(\boldsymbol{r}) = (\mu_o/4\pi)\int_{-\infty}^{\infty}[\boldsymbol{J}(\boldsymbol{r}')/|\boldsymbol{r}-\boldsymbol{r}'|]\mathrm{d}\boldsymbol{r}'$ to calculate the vector potential of the wire in its surrounding space.

Problem 3) Consider Maxwell's equations for a system in which $\rho_{\text{free}}(\boldsymbol{r},t)=0$ and $\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t)=0$. The remaining sources of the electromagnetic field, $\boldsymbol{P}(\boldsymbol{r},t)$ and $\boldsymbol{M}(\boldsymbol{r},t)$, are arbitrary functions of the space-time coordinates.

2 Pts a) Eliminate $\boldsymbol{E}$ and $\boldsymbol{B}$ from Maxwell's equations by expressing the sources in terms of bound magnetic charge and current densities, that is, $\rho^{(m)}_{\text{bound}}$ and $\boldsymbol{J}^{(m)}_{\text{bound}}$.

2 Pts b) Define a magnetic scalar potential $\psi^{(m)}(\boldsymbol{r},t)$ and a magnetic vector potential $\boldsymbol{A}^{(m)}(\boldsymbol{r},t)$ suitable for the Maxwell equations written in terms of $\boldsymbol{D}(\boldsymbol{r},t)$ and $\boldsymbol{H}(\boldsymbol{r},t)$ in part (a).

2 Pts c) What is the equivalent of the Lorenz gauge for the magnetic potentials defined in part (b)?

2 Pts d) In the aforementioned Lorenz-equivalent gauge, find the wave equation that relates $\psi^{(m)}(\boldsymbol{r},t)$ to $\rho^{(m)}_{\text{bound}}(\boldsymbol{r},t)$, and also the wave equation that relates $\boldsymbol{A}^{(m)}(\boldsymbol{r},t)$ to $\boldsymbol{J}^{(m)}_{\text{bound}}(\boldsymbol{r},t)$.

2 Pts e) Using the analogy between the equations obtained in part (d) and the standard wave equation, express the solutions $\psi^{(m)}(\boldsymbol{r},t)$ and $\boldsymbol{A}^{(m)}(\boldsymbol{r},t)$ as integrals over $\rho^{(m)}_{\text{bound}}(\boldsymbol{r},t)$ and $\boldsymbol{J}^{(m)}_{\text{bound}}(\boldsymbol{r},t)$.
