Opti 501 **1st Midterm Exam** (9/27/2018) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

Problem 1) A circularly polarized plane-wave of frequency ω propagates in free space along the direction of the x-axis. The electric and magnetic fields of the plane-wave are specified as

$$\boldsymbol{E}(\boldsymbol{r},t) = E_0 \cos[(\omega/c)x - \omega t]\,\hat{\boldsymbol{y}} + E_0 \sin[(\omega/c)x - \omega t]\,\hat{\boldsymbol{z}}.$$

$$\boldsymbol{H}(\boldsymbol{r},t) = (E_0/Z_0)\cos[(\omega/c)x - \omega t]\,\hat{\boldsymbol{z}} - (E_0/Z_0)\sin[(\omega/c)x - \omega t]\,\hat{\boldsymbol{y}}.$$

In the above equations, $c = (\mu_0\varepsilon_0)^{-½}$ is the speed of light in vacuum, while $Z_0 = (\mu_0/\varepsilon_0)^{½}$ is the impedance of free space. Assume that $\rho_{\text{free}}(\boldsymbol{r},t) = 0$, $\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t) = 0$, $\boldsymbol{P}(\boldsymbol{r},t) = 0$, $\boldsymbol{M}(\boldsymbol{r},t) = 0$.

5 pts a) Show that the specified $\boldsymbol{E}$ and $\boldsymbol{H}$ fields satisfy all four of Maxwell's equations.

3 pts b) Find the E-field energy-density, $\mathbb{E}(\boldsymbol{r},t) = ½\varepsilon_0\boldsymbol{E}\cdot\boldsymbol{E}$, the H-field energy-density, $\mathbb{H}(\boldsymbol{r},t) = ½\mu_0\boldsymbol{H}\cdot\boldsymbol{H}$, and the Poynting vector, $\boldsymbol{S}(\boldsymbol{r},t) = \boldsymbol{E}(\boldsymbol{r},t)\times\boldsymbol{H}(\boldsymbol{r},t)$, of the plane-wave.

3 pts c) Confirm the conservation of electromagnetic energy by demonstrating the validity of the Poynting theorem, $\boldsymbol{\nabla}\cdot\boldsymbol{S} + \partial_t(\mathbb{E}+\mathbb{H}) = 0$, as applied to the above plane-wave.

Hint: $\boldsymbol{\nabla}\cdot\boldsymbol{V} = \partial_x V_x + \partial_y V_y + \partial_z V_z$.

$\boldsymbol{\nabla}\times\boldsymbol{V} = (\partial_y V_z - \partial_z V_y)\hat{\boldsymbol{x}} + (\partial_z V_x - \partial_x V_z)\hat{\boldsymbol{y}} + (\partial_x V_y - \partial_y V_x)\hat{\boldsymbol{z}}.$

$\boldsymbol{A}\times\boldsymbol{B} = (A_y B_z - A_z B_y)\hat{\boldsymbol{x}} + (A_z B_x - A_x B_z)\hat{\boldsymbol{y}} + (A_x B_y - A_y B_x)\hat{\boldsymbol{z}}.$

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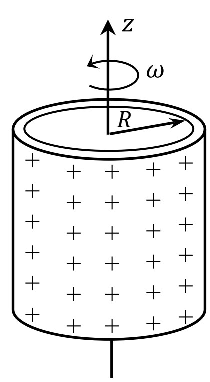
4 pts **Problem 2**) Shown in the figure below is a thin cylindrical shell of radius R and infinite length, having a constant uniform surface charge-density σ_{s0}, and rotating at the constant angular velocity ω around the z-axis. Both regions inside and outside the shell are free space. Describe Maxwell's boundary conditions at the cylinder's surface.



Hint: The boundary conditions relate the tangential and perpendicular components of the fields, namely, $\boldsymbol{D}_\perp$, $\boldsymbol{B}_\perp$, $\boldsymbol{E}_\parallel$ and $\boldsymbol{H}_\parallel$ immediately before the surface at $(\rho = R^-, \varphi, z, t)$ and immediately after the surface at $(\rho = R^+, \varphi, z, t)$ to each other and to the surface-charge and surface-current densities $\sigma_s(\boldsymbol{r},t)$ and $\boldsymbol{J}_s(\boldsymbol{r},t)$.

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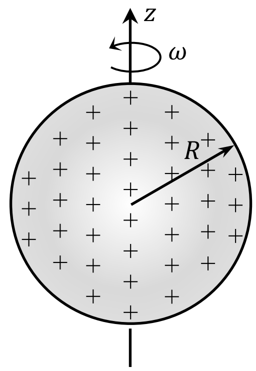
4 pts **Problem 3**) A thin spherical shell of radius R has uniform surface charge-density σ_{s0} and rotates at constant angular velocity ω around the z-axis, as shown. The region inside as well as that outside the shell is free space. Describe Maxwell's boundary conditions at the sphere's surface.



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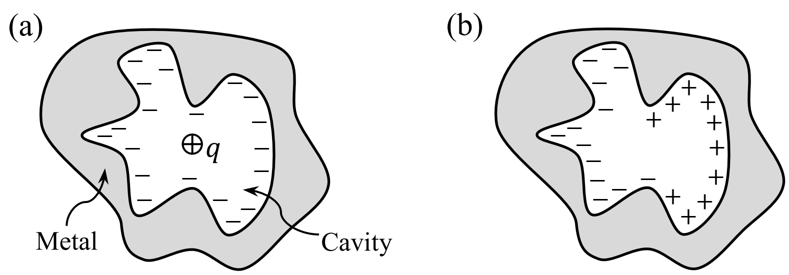
3 pts **Problem 4**) a) An electrically-charged particle of charge q resides within a cavity inside a metallic host, as shown in Fig.(a) below. Show that the total electric charge induced on the walls of the cavity must equal $-q$.



3 pts b) In accordance with the above argument, when the charge q is removed from the cavity, the *total* charge remaining on the cavity walls must amount to zero; see Fig.(b). Show that, in the absence of the charge q inside the cavity, no charges whatsoever can exist anywhere on the cavity walls.

Hint: In electrostatic problems, Maxwell's 1st equation $\oint_{\text{surface}} \boldsymbol{D}(\boldsymbol{r}) \cdot \mathrm{d}\boldsymbol{s} = \int_{\text{volume}} \rho_{\text{free}}(\boldsymbol{r})\mathrm{d}v$ as well as Maxwell's 3rd equation $\oint_{\text{loop}} \boldsymbol{E}(\boldsymbol{r}) \cdot \mathrm{d}\boldsymbol{\ell} = 0$ must be simultaneously satisfied.
