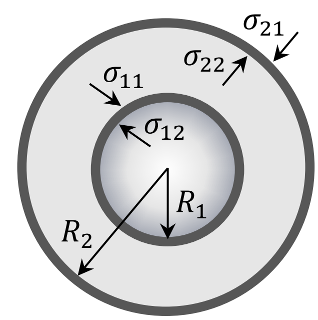
Opti 501 **1st Midterm Exam** (10/6/2016) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

Problem 1) A hollow, electrically conducting, spherical shell of radius R_1 and surface charge-density σ_1 resides within another hollow, electrically conducting, spherical shell of radius R_2 and surface-charge-density σ_2, as shown. The two spheres are concentric and have relatively thin shells.



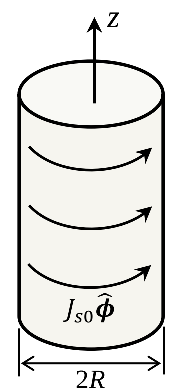
2 pts a) Determine the charge-densities σ_{11}, σ_{12}, σ_{21}, and σ_{22} on the inner and outer surfaces of each sphere.

2 pts b) Find the electric field distribution in the various regions inside and outside the spheres.

1 pt c) Assuming the total charge on one sphere is equal in magnitude and opposite in sign to the total charge on the other sphere, what is the capacitance C of the pair of spheres?

Hint: $C = Q/V$, where $\pm Q$ is the total charge on each sphere, and V is the potential difference (or voltage) between the spheres. The voltage is the integral of the E-field along the radial direction from one sphere to the other.

Problem 2) A hollow circular cylinder having infinite length, negligible thickness τ, and radius R, is aligned with the z-axis and carries a constant surface-current-density $J_{s0}\hat{\boldsymbol{\varphi}}$.



2 pts a) What is the relationship between the ordinary current-density $\boldsymbol{J}_{\text{free}}$ and a surface-current-density such as $J_{s0}\hat{\boldsymbol{\varphi}}$? What are the units of J_{s0}?

2 pts b) Use symmetry arguments to constrain the components H_ρ, H_φ, H_z of the magnetic field created both inside and outside the cylinder. The constraints may apply to the field components, but also to their dependence on the cylindrical coordinates ρ, φ and z.

3 pts c) Use Maxwell's equations to determine the dependence of H_ρ, H_φ, and H_z on J_{s0} and also on the relevant coordinate(s).

Problem 3) Four friends are discussing the nature of the sources of the electromagnetic field as revealed to them by Maxwell's equations. They all agree that $\rho_{\text{free}}(\boldsymbol{r}, t)$ and $\boldsymbol{J}_{\text{free}}(\boldsymbol{r}, t)$ are produced by (stationary or mobile) electrical charges. However, they vehemently disagree as to the nature of the remaining two sources, namely, polarization $\boldsymbol{P}(\boldsymbol{r}, t)$ and magnetization $\boldsymbol{M}(\boldsymbol{r}, t)$.

Alice: Polarization gives rise to electric charge-density $-\boldsymbol{\nabla} \cdot \boldsymbol{P}$ and electric current-density $\partial \boldsymbol{P}/\partial t$, whereas magnetization has no electric charge-density associated with it, only electric current-density, which is given by $\mu_0^{-1}\boldsymbol{\nabla} \times \boldsymbol{M}$.

Brian: I couldn't disagree more. There are no electric charges, nor electric currents, associated with either $\boldsymbol{P}$ or $\boldsymbol{M}$. Everything stems from pairs of magnetic charges (i.e., magnetic monopoles). Magnetization gives rise to the magnetic charge-density $-\boldsymbol{\nabla} \cdot \boldsymbol{M}$ and magnetic current-density

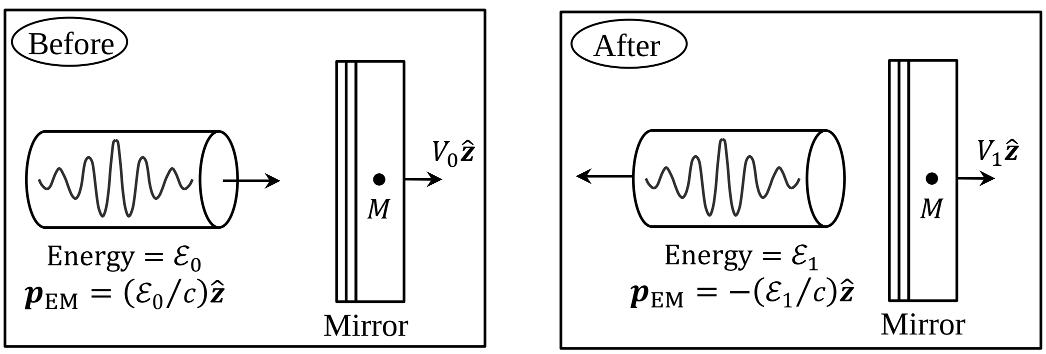
$\partial \boldsymbol{M}/\partial t$, whereas polarization has no magnetic charge-density associated with it, only magnetic current-density, which is given by $-\varepsilon_0^{-1}\boldsymbol{\nabla}\times\boldsymbol{P}$.

Carol: I agree with Alice about polarization, but I think Brian has it right when it comes to magnetization.

David: I am afraid I must disagree with Carol. In my opinion, Alice is correct about magnetization, while Brian has the right idea about polarization.

8 pts You be the judge. Who is right here and why? (You must explain your reasoning and summon support for your arguments directly and exclusively from Maxwell's equations.)

Problem 4) A light bullet carrying energy $\mathcal{E}_0$ and electromagnetic momentum $\boldsymbol{p}_{\text{EM}} = (\mathcal{E}_0/c)\hat{\boldsymbol{z}}$ travels in free space along the z-axis. The light pulse (i.e., the bullet) is reflected from a perfect reflector at normal incidence, as shown below. The reflector's mass is M and its initial velocity (i.e., velocity before interacting with the light pulse) is $V_0\hat{\boldsymbol{z}}$. After reflection, the light pulse has energy $\mathcal{E}_1$ and electromagnetic momentum $\boldsymbol{p}_{\text{EM}} = -(\mathcal{E}_1/c)\hat{\boldsymbol{z}}$, while the mirror's velocity has changed to $V_1\hat{\boldsymbol{z}}$.



1 pt a) Write the relativistic equations for the conservation of the overall energy and linear momentum of the system.

1 pt b) Write the non-relativistic equations for the conservation of the overall energy and linear momentum of the system. (In this case the mirror's energy is its kinetic energy $\mathcal{E}_K = ½MV^2$.)

2 pts c) Solve the non-relativistic equations of part (b) to obtain expressions for $\mathcal{E}_1$ and V_1 in terms of $\mathcal{E}_0$, V_0, M, and c.

1 pts d) Considering that, in the non-relativistic regime, $\mathcal{E}_0 \ll Mc^2$ and $V_0 \ll c$, use the approximation $\sqrt{1+\varepsilon} \cong 1 + ½\varepsilon - ⅛\varepsilon^2$ to arrive at simple (approximate) formulas for V_1 and $\mathcal{E}_1$.

Hint: You may define $\alpha_0 = \mathcal{E}_0/Mc^2$, $\alpha_1 = \mathcal{E}_1/Mc^2$, $\beta_0 = V_0/c$ and $\beta_1 = V_1/c$ to simplify algebraic manipulations.
