Opti 501 **1st Midterm Exam** (10/6/2015) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

Problem 1) A plane wave having wave-vector $\boldsymbol{k}$ and frequency ω propagates in free space. Let the electric and magnetic fields of the plane-wave be written as

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_0 \exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)],$$

$$\boldsymbol{H}(\boldsymbol{r},t) = \boldsymbol{H}_0 \exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)].$$

No sources are assumed to reside in free space; therefore, $\rho_{\text{free}}(\boldsymbol{r},t) = 0$, $\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t) = 0$, $\boldsymbol{P}(\boldsymbol{r},t) = 0$, and $\boldsymbol{M}(\boldsymbol{r},t) = 0$.

1 Pt a) What does Maxwell's *first* equation have to say about the relation between $\boldsymbol{E}_0$ and $\boldsymbol{k}$?

1 Pt b) What does Maxwell's *fourth* equation have to say about the relation between $\boldsymbol{H}_0$ and $\boldsymbol{k}$?

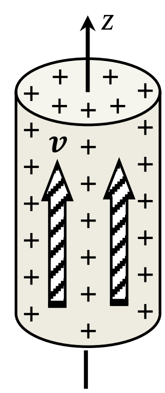
1 Pt c) Apply Maxwell's *second* equation to derive a relation connecting $\boldsymbol{E}_0$, $\boldsymbol{H}_0$, $\boldsymbol{k}$, and ω.

1 Pt d) Apply Maxwell's *third* equation to derive another relation connecting $\boldsymbol{E}_0$, $\boldsymbol{H}_0$, $\boldsymbol{k}$, and ω.

2 Pts e) Combine the results obtained in parts (a)-(d) to eliminate $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ from the equations, thus arriving at the connection between $\boldsymbol{k}$ and ω for plane-waves that travel in free space.

Hint: In part (e), the following vector identity will be helpful: $\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C}) = (\boldsymbol{A}\cdot\boldsymbol{C})\boldsymbol{B} - (\boldsymbol{A}\cdot\boldsymbol{B})\boldsymbol{C}$.

Problem 2) An infinitely-long, solid cylinder of radius R is uniformly filled with a constant volume charge-density ρ_0 (units: *coulomb/m*^{3}). The charges within the cylinder are moving up along the z-axis at a constant linear velocity $\boldsymbol{v} = v_0\hat{\boldsymbol{z}}$.



1 Pt a) Find the E-field distribution both inside and outside the cylinder.

1 Pt b) What is the density $\boldsymbol{J}$ of the electric current associated with the movement of the charges?

2 Pts c) Find the magnetic field distribution both inside and outside the cylinder.

2 Pts d) Calculate the force density (i.e., force per unit volume) exerted by the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields on the moving charge distribution. Compare the strength of these two forces. Under what circumstances will the (radially inward) pull of the magnetic force be equal to the (radially outward) push of the electric force?

Problem 3) Use your knowledge of Newtonian mechanics, Maxwell's equations, the Lorentz force law, and the definitions of work, force, energy, capacitance, etc., to express the units of the following entities in terms of the fundamental units of the *SI* (or MKSA) system, namely, meter (*m*), kilogram (*kg*), second (*s*), and ampere (*A*).

1 Pt	a) Polarization:	$\boldsymbol{P}$	[*coulomb*/*m*^{2}]
1 Pt	b) Magnetization:	$\boldsymbol{M}$	[*weber*/*m*^{2}]
1 Pt	c) Electrical resistance:	R	[*ohm*]

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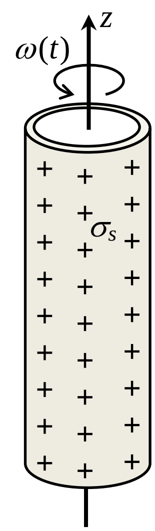
1 Pt d) Capacitance: C [*farad*]

1 Pt e) Inductance: L [*henry*]

1 Pt f) Momentum density: $\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}/c^2$ [$volt \cdot ampere \cdot sec^2/m^4$]

Problem 4) An infinitely-long, hollow cylinder of radius R and negligible thickness has a uniform electric charge-density σ_s (units: $coulomb/m^2$) on its surface. The cylinder is slowly rotated from its initial stationary state, until, after a long time, it reaches the constant angular velocity ω_0.



1 Pt a) Find the electric field distribution inside and outside the cylinder. (At this stage you may ignore the E-field produced by the time-dependence of the H-field.)

1 Pt b) Assuming that the magnetic field *outside* the cylinder remains negligible at all times, determine the magnetic field distribution *inside* the cylinder.

1 Pt c) How much energy per unit length of the cylinder is stored in the magnetic field?

1 Pt d) Determine the electric field (induced by the time-dependence of the magnetic field inside the cylinder) that acts on surface charges to oppose the acceleration of the cylinder while it is spinning up.

2 Pts e) How much mechanical energy (per unit length) is spent during the spin-up process?

1 Pt f) What is the relation between the mechanical energy spent during the spin-up process, and the magnetic field energy stored inside the cylinder?
