Opti 501 **1^{st} Midterm Exam** (9/25/2014) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together. Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

Problem 1) Let $\boldsymbol{\alpha}=\boldsymbol{\alpha}'+\mathrm{i}\boldsymbol{\alpha}''$, $\boldsymbol{\beta}=\boldsymbol{\beta}'+\mathrm{i}\boldsymbol{\beta}''$, and $\boldsymbol{\gamma}=\boldsymbol{\gamma}'+\mathrm{i}\boldsymbol{\gamma}''$ be three arbitrary complex-valued vectors. Identify the real and imaginary components of the following vector-products:

1 Pt a) $\boldsymbol{\alpha}\cdot\boldsymbol{\beta}$.

1 Pt b) $\boldsymbol{\alpha}\times\boldsymbol{\beta}$.

1 Pt c) $\boldsymbol{\gamma}\times\boldsymbol{\gamma}$.

1 Pt d) $\boldsymbol{\alpha}\cdot(\boldsymbol{\beta}\times\boldsymbol{\gamma})$.

Problem 2) Let $\psi(\boldsymbol{r},t)=\psi_0\exp[\mathrm{i}(\boldsymbol{k}_1\cdot\boldsymbol{r}-\omega_1 t)]$ be a scalar plane-wave, and let $\boldsymbol{A}(\boldsymbol{r},t)=\boldsymbol{A}_0\exp[\mathrm{i}(\boldsymbol{k}_2\cdot\boldsymbol{r}-\omega_2 t)]$ and $\boldsymbol{B}(\boldsymbol{r},t)=\boldsymbol{B}_0\exp[\mathrm{i}(\boldsymbol{k}_3\cdot\boldsymbol{r}-\omega_3 t)]$ be two vector plane-waves. In general, ψ_0, ω_1, ω_2, and ω_3 are arbitrary complex constants, while $\boldsymbol{A}_0$, $\boldsymbol{B}_0$, $\boldsymbol{k}_1$, $\boldsymbol{k}_2$, and $\boldsymbol{k}_3$ are arbitrary complex vectors. Show that the above plane-waves satisfy the following vector identities:

2 Pts a) $\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\psi)=0$.

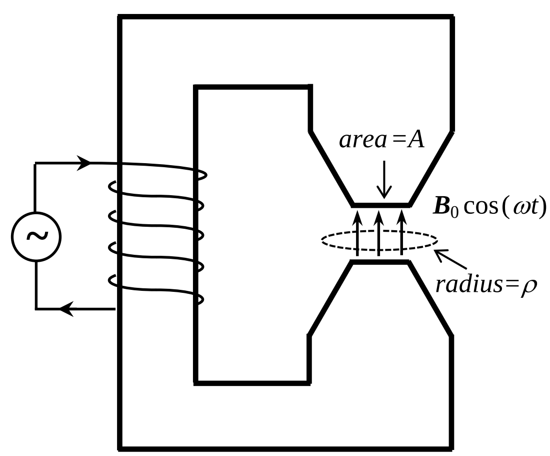
2 Pts b) $\boldsymbol{\nabla}\cdot(\psi\boldsymbol{A})=(\boldsymbol{\nabla}\psi)\cdot\boldsymbol{A}+\psi(\boldsymbol{\nabla}\cdot\boldsymbol{A})$.

2 Pts c) $\boldsymbol{\nabla}\times(\psi\boldsymbol{B})=(\boldsymbol{\nabla}\psi)\times\boldsymbol{B}+\psi\boldsymbol{\nabla}\times\boldsymbol{B}$.

2 Pts d) $\boldsymbol{\nabla}\cdot(\boldsymbol{A}\times\boldsymbol{B})=\boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{A})-\boldsymbol{A}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})$.

Hint: $\boldsymbol{U}\cdot(\boldsymbol{V}\times\boldsymbol{W})=\boldsymbol{V}\cdot(\boldsymbol{W}\times\boldsymbol{U})=\boldsymbol{W}\cdot(\boldsymbol{U}\times\boldsymbol{V})$.

Problem 3) An electromagnet is driven by a current source at a low frequency ω, so that the uniform magnetic field produced in the gap between the magnet's pole-pieces is given by $\boldsymbol{B}(\boldsymbol{r},t)=B_0\cos(\omega t)\,\hat{\boldsymbol{z}}$, as shown in the figure. The cross-sectional areas of both pole-pieces at the gap are A, and the gap-width is small compared to the dimensions of the pole-pieces, so that the magnetic field is fairly uniform and confined to the gap region.



2 Pts a) Determine the magnetic flux $\Phi(t)$ that crosses the dashed circle shown in the figure.

3 Pts b) Use the integral form of Maxwell's 3^{rd} equation (i.e., Faraday's law) to determine the E-field on the (dashed) circular loop of radius ρ that surrounds the gap. (Assume circular symmetry.)

3 Pts c) If the imagined dashed circle shown in the figure is replaced by a physical loop with electrical resistance R, what would be the induced electric current $I(t)$ in the loop?

Problem 4) Use your knowledge of Newtonian mechanics, Maxwell's equations, the Lorentz force law, and the definitions of work, force, energy, capacitance, etc., to express the units of the following entities in terms of the fundamental units of the *SI* (or MKSA) system, namely, meter (*m*), kilogram (*kg*), second (*s*), and ampere (*A*).

1 Pt	a)	Electric field:	$\boldsymbol{E}$	[*volt*/*meter*].
1 Pt	b)	Magnetic induction:	$\boldsymbol{B}$	[$weber/m^2$].
1 Pt	c)	Poynting vector:	$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}$	[$volt \cdot ampere/m^2$].
1 Pt	d)	Permittivity of free space:	ε_0	[*farad*/*meter*].
1 Pt	e)	Permeability of free space:	μ_0	[*henry*/*meter*].