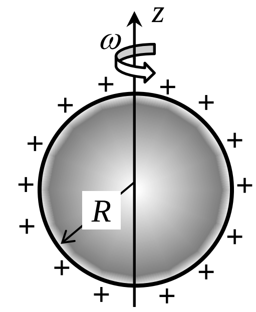
Opti 501 **1st Midterm Exam** (10/1/2013) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

Note: Bold symbols represent vectors and vector fields.



Problem 1) A spherical shell of radius R has a constant, uniform surface-charge-density σ_s [coulomb/m^2]. The shell rotates around the z-axis at a constant angular velocity ω.

2 Pts a) Write an expression for the surface-current-density $\boldsymbol{J}_s(\rho, \theta, \phi, t)$ in the spherical coordinate system and specify the units of $\boldsymbol{J}_s$.

2 Pts b) Compute the divergence of $\boldsymbol{J}_s$ and confirm that the charge-current continuity equation $\boldsymbol{\nabla} \cdot \boldsymbol{J}_s + \partial \sigma_s / \partial t = 0$ is satisfied.

Problem 2) A vector plane-wave propagating in free space has the following E-field amplitude:

$$\boldsymbol{E}(\boldsymbol{r}, t) = \text{Real}\{\boldsymbol{E}_0 \exp[i(\boldsymbol{k} \cdot \boldsymbol{r} - \omega t)]\}.$$

Here ω is a real-valued scalar parameter, while $\boldsymbol{k} = \boldsymbol{k}' + i\boldsymbol{k}''$ and $\boldsymbol{E}_0 = \boldsymbol{E}_0' + i\boldsymbol{E}_0''$ are complex-valued vectors.

1 Pt a) What is the oscillation frequency of the E-field (when considered as a function of time)?

2 Pts b) At what rate and in which direction does the field amplitude decay with distance from the origin?

3 Pts c) Identify planes of constant phase in the three-dimensional Euclidean space whose individual points in Cartesian coordinates are specified by $\boldsymbol{r} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}$. Find the relationship between the phase-velocity and the various parameters of the plane-wave.

2 Pts d) Which parameter(s) of the plane-wave specify its state of polarization? Under what circumstances is the plane-wave linearly polarized? When is the beam circularly polarized?

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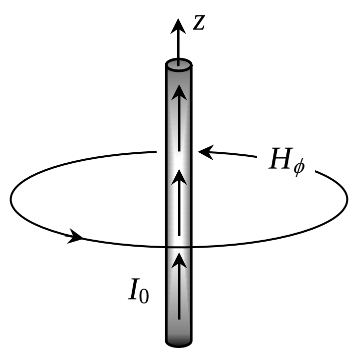
4 Pts **Problem 3**) An infinitely long, thin wire carries the constant current I_0 along the z-axis. Using symmetry arguments in conjunction with Maxwell's equations, explain why the H-field produced by the wire *cannot* have a component along the radial direction. In other words, explain why $H_\rho(\rho, \phi, z)$ is zero everywhere in the space surrounding the wire.



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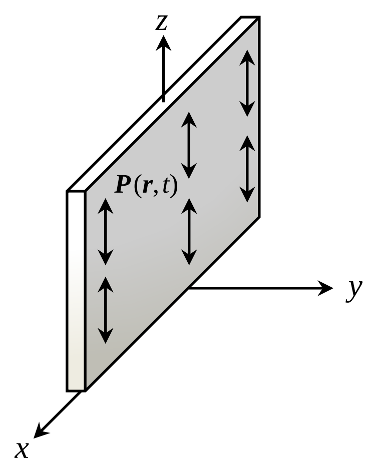
3 Pts **Problem 4**) a) The polarization of a thin sheet of material located in the xz-plane is given by

$$\boldsymbol{P}(\boldsymbol{r},t) = P_0\hat{\boldsymbol{z}}\,\delta(y)\sin(\kappa z - \omega t).$$

Here P_0, κ and ω are real-valued constants, and $\delta(\cdot)$ is Dirac's delta function. Find the densities of bound electric charge $\rho_{\text{bound}}^{(\text{e})}(\boldsymbol{r},t)$ and bound electric current $\boldsymbol{J}_{\text{bound}}^{(\text{e})}(\boldsymbol{r},t)$ within the sheet. Show that the charge-current continuity equation is satisfied.



2 Pts b) The magnetization of a thin sheet of material located in the xz-plane is given by

$$\boldsymbol{M}(\boldsymbol{r},t) = M_0\hat{\boldsymbol{y}}\,\delta(y)\cos(\kappa z - \omega t).$$

As before, M_0, κ and ω are real-valued constants, and $\delta(\cdot)$ is Dirac's delta function. Find the densities of bound electric charge $\rho_{\text{bound}}^{(\text{e})}(\boldsymbol{r},t)$ and bound electric current $\boldsymbol{J}_{\text{bound}}^{(\text{e})}(\boldsymbol{r},t)$ within the sheet. Show that the charge-current continuity equation is satisfied.

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4 Pts **Problem 5**) An infinitely long, thin wire has a uniform charge-density λ_0 [coulomb/meter]. The wire is centered on the z-axis and surrounded by a cylindrical shell of an electrically conducting material. The shell is infinitely long in the z-direction, is centered on the z-axis, and its inner and outer radii are R_1 and R_2, respectively. Assuming that the cylindrical shell is *not* charged at the outset, and considering that the E-field everywhere inside the metallic conductor must vanish, find the surface charge densities σ_1 and σ_2 at the inner and outer surfaces of the cylindrical shell.