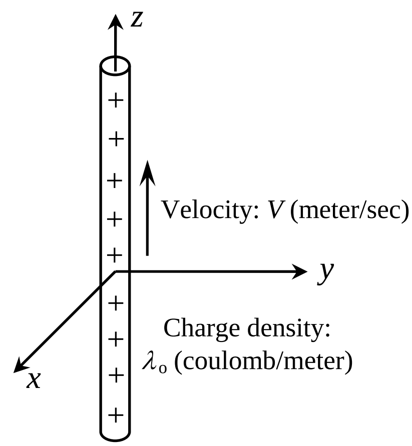
Opti 501 **1^{st} Midterm Exam** (10/2/2012) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

Note: Bold symbols represent vectors and vector fields.



Problem 1) An infinitely-long, thin rod has a uniform charge density λ_o along the z-axis. The rod also moves at a constant velocity V in the $+z$ direction.

2 Pts a) Determine the constant current I_o flowing along the z-axis.

2 Pts b) Use symmetry arguments in conjunction with Maxwell's equations to determine the E- and H-field distributions in the rod's surrounding space.

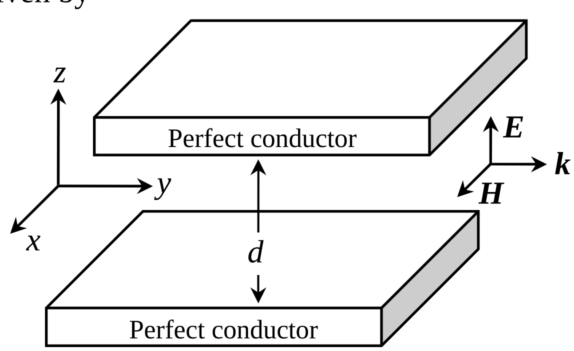
2 Pts c) Show that the solution obtained in part (b) above satisfies all four Maxwell's equations.

2 Pts d) Determine the rate of flow of electromagnetic energy (per unit area per unit time) in the vicinity of the rod.

Problem 2) In the free space region between two infinitely large, perfectly conducting parallel plates, an electromagnetic plane-wave propagates along the y-axis, as shown in the figure. The electric and magnetic fields of the plane-wave are given by

$$\boldsymbol{E}(\boldsymbol{r},t) = E_o \cos(k_o y - \omega_o t)\hat{\boldsymbol{z}},$$

$$\boldsymbol{H}(\boldsymbol{r},t) = H_o \cos(k_o y - \omega_o t)\hat{\boldsymbol{x}},$$



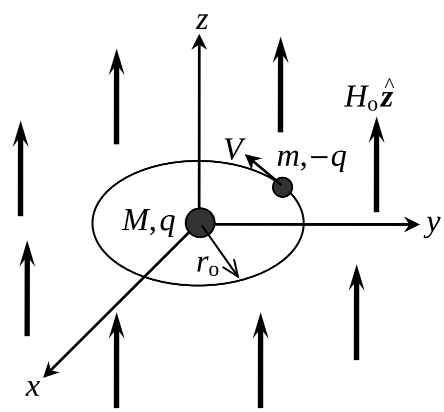
where E_o and H_o are the field amplitudes, k_o is the propagation constant (also known as the wave-number), and ω_o is the angular frequency of the plane-wave. All the above parameters, i.e., E_o, H_o, k_o and ω_o, are real-valued constants.

4 Pts a) Write Maxwell's equations in the region between the plates (i.e., $-\tfrac{1}{2}d < z < \tfrac{1}{2}d$), then find the relationship between k_o and ω_o on the one hand, and that between E_o and H_o on the other. (**Hint**: These relations should involve the speed of light in vacuum, $c = 1/\sqrt{\mu_o \varepsilon_o}$, and the impedance of free space, $Z_o = \sqrt{\mu_o/\varepsilon_o}$).

3 Pts b) Considering that the E and H fields inside the (perfectly conducting) plates must be zero, use Maxwell's boundary conditions to find the surface charge-density $\sigma_s(x,y,z = \pm\tfrac{1}{2}d, t)$ and the surface current-density $\boldsymbol{J}_s(x,y,z = \pm\tfrac{1}{2}d, t)$ on the inner surfaces of both plates.

2 Pts c) Show that the surface charge and current densities obtained in part (b) satisfy the charge-current continuity equation.

Problem 3) A positive point-charge of mass M and charge q sits at the origin of a coordinate system, while a negative point-charge of mass m and charge $-q$ revolves around the origin in a circular orbit of radius r_o. A constant, uniform magnetic field $H_o\hat{\boldsymbol{z}}$ is applied perpendicular to the plane of the orbit. We assume $M >> m$, so that the central charge may be considered stationary. We also assume the velocity V of the rotating charge to be well below the speed of light, c, so that relativistic effects may be ignored.



2 Pts a) With respect to the origin of coordinates, what is the orbital angular momentum $\mathcal{L}$ of the rotating particle in terms of m, V and r_o?

2 Pts b) Write an expression for the magnetic dipole moment $\boldsymbol{m}$ of the system in terms of q, V and r_o, then proceed to express $\boldsymbol{m}$ in terms of the angular momentum $\mathcal{L}$ of the rotating particle.

2 Pts c) Write an expression for the electromagnetic force $\boldsymbol{F}$ experienced by the revolving charged particle in the presence of the central (stationary and positive) point-charge q and the externally applied magnetic field $H_o\hat{\boldsymbol{z}}$.

2 Pts d) Using Newton's law $\boldsymbol{F} = \mathrm{d}\boldsymbol{p}/\mathrm{d}t$, which relates the force $\boldsymbol{F}$ exerted on an object to the time-rate-of-change of its linear momentum $\boldsymbol{p}$, write the equation of motion of the revolving charge, relating its radius r_o and velocity V to the system parameters m, q and H_o, as well as to the permittivity ε_o and permeability μ_o of free space.
