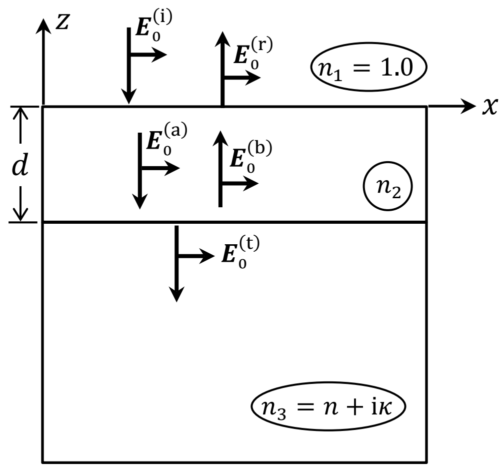
Opti 501 **Final Exam** (12/13/2016) **Time: 2 hours**

Please write your name and ID number on all the pages, then staple them together. Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

Problem 1) A homogeneous plane-wave of frequency ω is normally incident on a dielectric layer of thickness d and refractive index n_2, which is coated on a semi-infinite substrate of (complex) refractive index $n_3 = n + \mathrm{i}\kappa$. The incidence medium is free space, and $\mu(\omega) = 1$ for the dielectric layer and also for its substrate. Let us denote by $\rho_{\ell m}$ and $\tau_{\ell m}$ the Fresnel reflection and transmission coefficients at the interface between the incidence medium ℓ and the transmission medium m.



2 Pts a) Write expressions for the Fresnel reflection and transmission coefficients ρ_{12}, τ_{12}, ρ_{21}, τ_{21}, ρ_{23}, τ_{23} at normal incidence.

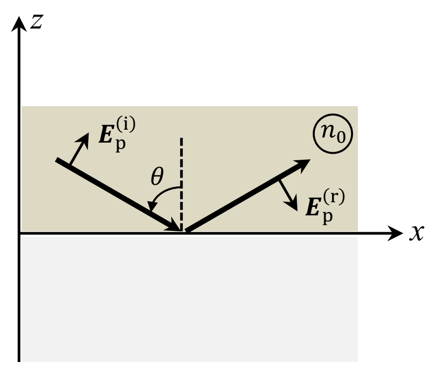
4 Pts b) Write a self-consistency equation for the E-field amplitude $E_0^{(\mathrm{a})}$ of the downward propagating plane-wave inside the dielectric layer. Solve the equation to obtain an expression for $E_0^{(\mathrm{a})}/E_0^{(\mathrm{i})}$.

2 Pts c) Write an expression relating $E_0^{(\mathrm{t})}$ to $E_0^{(\mathrm{a})}$, then determine the overall transmission coefficient $E_0^{(\mathrm{t})}/E_0^{(\mathrm{i})}$.

2 Pts d) Write an expression relating $E_0^{(\mathrm{r})}$ to $E_0^{(\mathrm{i})}$ and $E_0^{(\mathrm{a})}$, then determine the overall reflection coefficient $E_0^{(\mathrm{r})}/E_0^{(\mathrm{i})}$.

2 Pts e) Show that the dielectric layer will have no effect on the reflectivity of the substrate if the layer thickness d happens to be an integer-multiple of half wavelength, namely, $d = m\lambda_0/(2n_2)$, where m is an arbitrary integer and $\lambda_0 = 2\pi c/\omega$ is the vacuum wavelength.

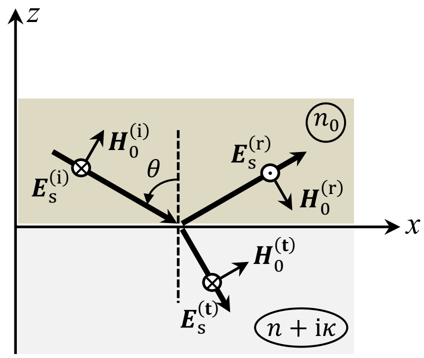
Problem 2) A homogeneous plane-wave of frequency ω propagates inside a linear, homogeneous, isotropic, transparent, non-magnetic medium of refractive index $n_0(\omega)$. The beam arrives at the flat interface with a second linear, isotropic, homogeneous medium. The incident beam is p-polarized with E-field amplitude $\boldsymbol{E}_\mathrm{p}^{(\mathrm{i})}$, the incidence angle is θ, and the Fresnel reflection coefficient at the interface is $\rho_\mathrm{p} = |\rho_\mathrm{p}| \exp(\mathrm{i}\varphi_\mathrm{p})$.



4 Pts a) Write expressions for the incident and reflected k-vectors, E-fields, and H-fields in the region $z > 0$.

4 Pts b) Find the time-averaged Poynting vector $\langle \boldsymbol{S}(\boldsymbol{r}, t) \rangle$ for both incident and reflected beams in terms of $E_\mathrm{p}^{(\mathrm{i})}$, ρ_p, n_0, and θ.

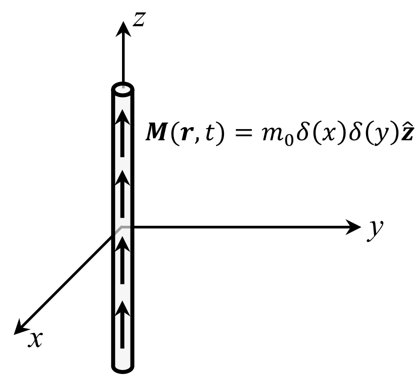
Problem 3) A homogeneous plane-wave of frequency ω propagating within a linear, homogeneous, isotropic, transparent, medium of refractive index n_0 arrives at the flat interface with a second linear, isotropic, homogeneous, non-magnetic medium of (complex) refractive index $n + \mathrm{i}\kappa$. The incident beam is s-polarized with E-field amplitude $\boldsymbol{E}_\mathrm{s}^{(\mathrm{i})}$, the incidence angle is θ, and the Fresnel transmission coefficient at the interface is $\tau_\mathrm{s} = |\tau_\mathrm{s}| \exp(\mathrm{i}\varphi_\mathrm{s})$.



4 Pts a) Write expressions for the transmitted k-vector, E-field, and H-field in the region $z < 0$.

4 Pts b) Find the time-averaged Poynting vector $\langle \boldsymbol{S}(\boldsymbol{r}, t) \rangle$ for the transmitted beam in terms of $E_\mathrm{s}^{(\mathrm{i})}$, τ_s, n_0, n, κ, and θ.

Problem 4) An infinitely long, thin wire aligned with the z-axis is uniformly magnetized along z. The magnetic dipole moment per unit-length of the wire is specified as $m_0\hat{\boldsymbol{z}}$.



4 Pts a) Find the bound electric charge-density $\rho_\mathrm{bound}^{(e)}(\boldsymbol{r}, t)$ and the bound electric current-density $\boldsymbol{J}_\mathrm{bound}^{(e)}(\boldsymbol{r}, t)$ of the wire.

4 Pts b) Determine the scalar and vector potentials of the wire in the surrounding space, using the following standard (Lorenz gauge) formulas:

$$\psi(\boldsymbol{r}, t) = \frac{1}{4\pi\varepsilon_0} \iiint_{-\infty}^{\infty} \frac{\rho_\mathrm{total}^{(e)}(\boldsymbol{r}', \; t - |\boldsymbol{r} - \boldsymbol{r}'|/c)}{|\boldsymbol{r} - \boldsymbol{r}'|} \mathrm{d}\boldsymbol{r}',$$

$$\boldsymbol{A}(\boldsymbol{r}, t) = \frac{\mu_0}{4\pi} \iiint_{-\infty}^{\infty} \frac{\boldsymbol{J}_\mathrm{total}^{(e)}(\boldsymbol{r}', \; t - |\boldsymbol{r} - \boldsymbol{r}'|/c)}{|\boldsymbol{r} - \boldsymbol{r}'|} \mathrm{d}\boldsymbol{r}'.$$

4 Pts c) Find the electric and magnetic field distributions, $\boldsymbol{E}(\boldsymbol{r}, t)$ and $\boldsymbol{H}(\boldsymbol{r}, t)$, in the space surrounding the wire.

Hint: $\displaystyle\int \frac{\mathrm{d}z}{(\alpha + z^2)^{3/2}} = \frac{z}{\alpha\sqrt{\alpha + z^2}}$, (G&R **2.264**-5).
