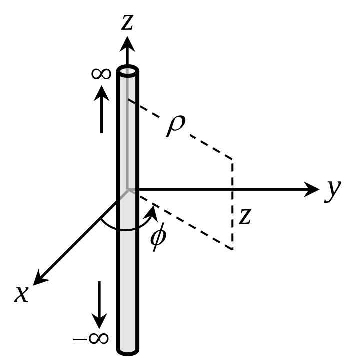
Opti 501 **Final Exam** (12/17/2014) **Time: 2 hours**

Please write your name and ID number on all the pages, then staple them together. Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

Problem 1) An infinitely long, thin wire along the z-axis has a constant and uniform electric dipole-moment $p_0\hat{\boldsymbol{z}}$ per unit-length.



3 Pts a) Use Dirac's δ-function notation to express the polarization density $\boldsymbol{P}(\boldsymbol{r},t)$ of the wire.

3 Pts b) Find the scalar and vector potentials established by the polarized wire in its surrounding space.

2 Pts c) Determine the electric and magnetic fields of the wire in its surrounding space.

5 Pts d) Repeat parts (a)–(c) for an infinitely long, thin, uniformly-magnetized wire along the z-axis, whose magnetic dipole-moment per unit-length of the wire is $m_0\hat{\boldsymbol{z}}$.

Hint: $\int_0^{2\pi}\cos\varphi\exp(\mathrm{i}x\cos\varphi)\,d\varphi = \mathrm{i}2\pi J_1(x)$; $\int_0^{\infty} J_1(x)dx = 1$; $\int\frac{dx}{(1+x^2)^{3/2}} = \frac{x}{\sqrt{1+x^2}}$.

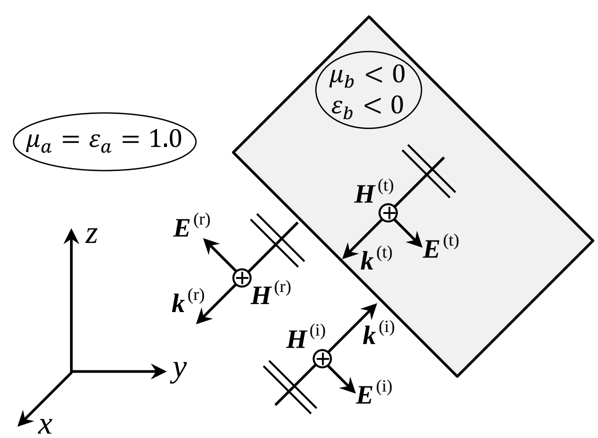
Problem 2) In the so-called "negative-index" media, the relative permittivity $\varepsilon(\omega)$ and the relative permeability $\mu(\omega)$ are both real-valued and negative in some range of frequencies such as the interval $\omega_1 \le \omega \le \omega_2$. Consider a homogeneous plane-wave of frequency ω and (real-valued) k-vector propagating in a linear, isotropic, homogeneous, negative-index medium. In this problem you are asked to examine the various properties of the plane-wave and, in particular, to show that its energy-flow direction is opposite to its k-vector.

3 Pts a) Use the dispersion relation to express the magnitude of the k-vector in terms of the frequency ω, the vacuum speed of light, c, and the permittivity and permeability of the negative-index medium at the oscillation frequency ω.

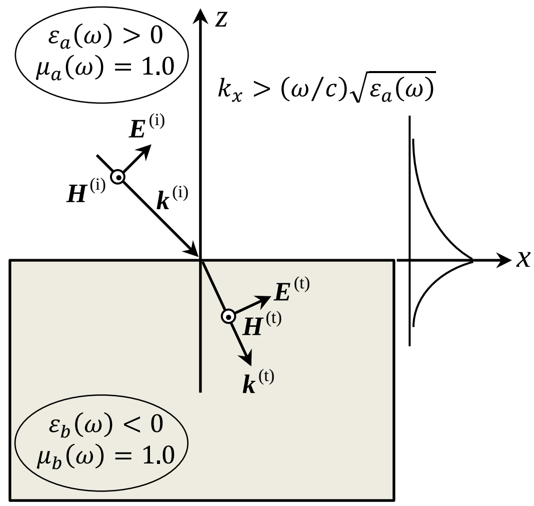
3 Pts b) Assuming the plane-wave's complex E-field amplitude is $\boldsymbol{E}_0$, use Maxwell's third equation (i.e., Faraday's law) to determine its H-field amplitude $\boldsymbol{H}_0$.

3 Pts c) Calculate the time-averaged Poynting vector $\langle\boldsymbol{S}(\boldsymbol{r},t)\rangle$ of the plane-wave, and show that its direction is opposite to that of the k-vector.

4 Pts d) Find the Fresnel reflection coefficient for a homogeneous plane-wave of frequency ω and wave-vector $\boldsymbol{k} = (\omega/c)\hat{\boldsymbol{k}}$, upon normal incidence from free space onto a negative-index medium having $\mu(\omega) = \varepsilon(\omega) = -1$. Specify the plane-wave that is thus transmitted into the negative-index medium.



Problem 3) Consider the interface between a semi-infinite transparent dielectric having a real-valued and positive permittivity, $\varepsilon_a(\omega) > 0$, and a semi-infinite lossless metallic medium having a real-valued and negative permittivity, $\varepsilon_b(\omega) < 0$. It is further assumed that $|\varepsilon_b| > \varepsilon_a$. The magnetic permeability for both media at the optical frequency ω may be set to unity, that is, $\mu_a(\omega) = \mu_b(\omega) = 1$. Let an *evanescent* plane-wave be incident from the transparent dielectric onto the metallic surface, thus transmitting an inhomogeneous plane-wave into the metallic medium. In this problem you are asked to investigate the viability of such a system in the *absence* of a third (i.e., reflected) plane-wave.



6 Pts a) Let the incident wave's frequency and k-vector be ω and $\boldsymbol{k}^{(\mathrm{i})} = k_x\hat{\boldsymbol{x}} + k_z^{(\mathrm{i})}\hat{\boldsymbol{z}}$, where k_x is real-valued and positive. Moreover, the incident wave is p-polarized, i.e., $\boldsymbol{E}_0^{(\mathrm{i})} = E_{x0}^{(\mathrm{i})}\hat{\boldsymbol{x}} + E_{z0}^{(\mathrm{i})}\hat{\boldsymbol{z}}$. Find the electric and magnetic fields of the wave transmitted into the metallic medium by matching the boundary conditions at the interfacial xy-plane at $z = 0$.

5 Pts b) Find the time-averaged Poynting vector $\langle \boldsymbol{S}(\boldsymbol{r}, t)\rangle$ for both plane-waves, i.e., the incident wave within the transparent dielectric, and the transmitted wave within the metallic medium.

3 Pts c) Repeat the same calculations as in part (a) for an s-polarized incident evanescent plane-wave, and show that, under the circumstances, the boundary conditions *cannot* be satisfied.
