Opti 501 **Final Exam** (12/19/2013) **Time: 2 hours**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

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2 Pts **Problem 1**) a) Write Maxwell's macroscopic equations for the static case, i.e., when the sources and the fields are time-independent.

2 Pts b) Show that the equations naturally split into two independent sets, one that describes electrostatics and another that describes magnetostatics.

2 Pts c) Identify the sources and the fields in the electrostatic case.

2 Pts d) Identify the sources and the fields in the magnetostatic case.

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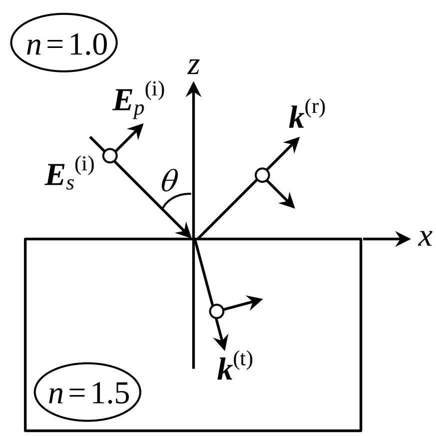
4 Pts **Problem 2**) a) Write Maxwell's macroscopic equations such that all the sources $(\rho_{\text{free}}, \boldsymbol{J}_{\text{free}}, \boldsymbol{P}, \boldsymbol{M})$ are expressed as free charge-density, free-current-density, bound *magnetic* charge-density, and bound *magnetic* current-density.

4 Pts b) Confirm the charge-current continuity equation for the bound magnetic charge and current densities, namely, $\boldsymbol{\nabla} \cdot \boldsymbol{J}_{\text{bound}}^{(\text{m})} + \partial \rho_{\text{bound}}^{(\text{m})} / \partial t = 0$.

Problem 3) A circularly-polarized, homogeneous plane-wave of frequency ω propagates in free space and arrives at the flat surface of a semi-infinite dielectric medium of refractive index $n(\omega) = 1.5$. The plane of incidence is xz, the dielectric surface is in the xy-plane at $z = 0$, the incidence angle is $\theta = 45°$, and the permeability of the dielectric medium is $\mu(\omega) = 1.0$.



2 Pts a) Find the reflectivity of the dielectric medium, that is, the fraction of the incident optical power that is reflected from the surface.

2 Pts b) What is the state of polarization of the reflected beam?

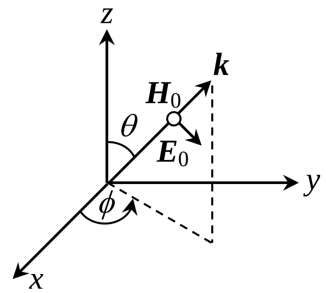
2 Pts c) Find the Fresnel transmission coefficients for both *p*- and *s*-components of the incident beam.

2 Pts d) What is the state of polarization of the transmitted beam?

2 Pts e) What fraction of the incident optical power is transmitted into the dielectric medium?

2 Pts f) Confirm that the total reflected and transmitted optical power equals that of the incident beam.

Problem 4) A homogeneous plane-wave of frequency ω propagates in free-space along a direction that has polar angle θ and azimuthal angle ϕ in the xyz coordinate system, as shown.



2 Pts a) Write general expressions for the k-vector, the E-field, and the H-field of the beam.

3 Pts b) Write all the relevant Maxwell's equations to specify as many components of the E- and H-fields as possible.

3 Pts c) Find the Poynting vector $\boldsymbol{S}$ as a function of position $\boldsymbol{r}$ (in 3-dimensional space) and time t. [**Note**: This is the time-dependent Poynting vector, *not* its time-averaged value $\langle \boldsymbol{S}(\boldsymbol{r}, t) \rangle$.]

2 Pts d) What is the density of the electromagnetic momentum $\boldsymbol{p}$ as a function of $(\boldsymbol{r}, t)$?

1 Pt e) Write expressions for the energy densities of the electric and magnetic fields. (You may simplify this part of the problem by assuming that $\boldsymbol{E}_0$ is real-valued, i.e., $\boldsymbol{E}_0'' = 0$, where $\boldsymbol{E}_0 = \boldsymbol{E}_0' + i\boldsymbol{E}_0''$.)

1 Pt f) Confirm the continuity of energy flux by verifying Poynting's theorem for the Poynting vector derived in part (c) and the energy densities derived in part (e).

Hint: $\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{A} \cdot \boldsymbol{C})\boldsymbol{B} - (\boldsymbol{A} \cdot \boldsymbol{B})\boldsymbol{C}$,

$(\boldsymbol{A} \times \boldsymbol{B}) \cdot (\boldsymbol{C} \times \boldsymbol{D}) = (\boldsymbol{A} \cdot \boldsymbol{C})(\boldsymbol{B} \cdot \boldsymbol{D}) - (\boldsymbol{A} \cdot \boldsymbol{D})(\boldsymbol{B} \cdot \boldsymbol{C})$,

$\sin^2 x = ½(1 - \cos 2x)$,

$\cos^2 x = ½(1 + \cos 2x)$,

$\sin x \cos x = ½ \sin 2x$.