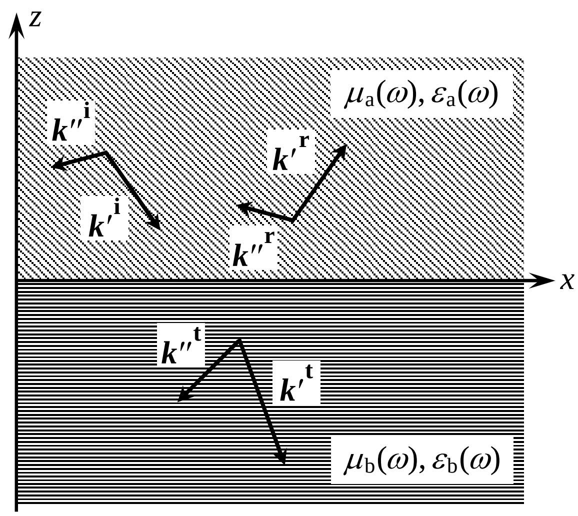
Opti 501 **Final Exam** (12/12/2012) **Time: 2 hours**

Please write your name and ID number on all the pages, then staple them together. Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

Problem 1) The Fresnel reflection and transmission coefficients (ρ_p, τ_p, ρ_s, τ_s) may be derived using Maxwell's equations with the aid of boundary conditions involving some components of the tangential $\boldsymbol{E}$ and $\boldsymbol{H}$ fields as well as the perpendicular components of the $\boldsymbol{D}$ and $\boldsymbol{B}$ fields at the interface between two media. Shown in the figure are two isotropic, linear, homogeneous, semi-infinite media joined at the xy-plane at $z=0$. The plane of incidence is xz, implying that the y-components of the incident, reflected, and transmitted k-vectors are all equal to zero.

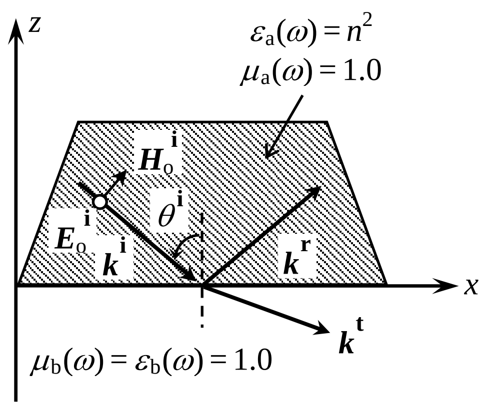


4 Pts a) Write all the relations among the various components of the k-vectors, the E-fields and the H-fields that can be derived from Maxwell's equations within each of the two media. (Treat the cases of p- and s-polarization separately.)

4 Pts b) Use the continuity of E_x and D_z at the interface to derive expressions for ρ_p and τ_p.

4 Pts c) Use the continuity of H_x and B_z at the interface to derive expressions for ρ_s and τ_s.

Problem 2) Inside a transparent dielectric prism of refractive index n, A monochromatic plane-wave of frequency ω propagates along the k-vector $\boldsymbol{k}^{\mathrm{i}} = k_x\hat{\boldsymbol{x}} + k_z^{\mathrm{i}}\hat{\boldsymbol{z}}$, as shown. The plane-wave is linearly polarized along the y-axis (i.e., s-polarization). The incidence angle θ^{i} is greater than the critical angle $\theta_c = \sin^{-1}(1/n)$ of total internal reflection.



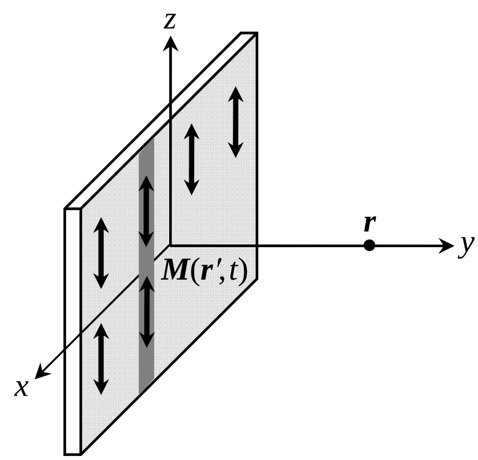
3 Pts a) Write complete expressions for the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields in the free-space region below the prism.

5 Pts b) Calculate the electromagnetic energy density $\mathcal{E}(\boldsymbol{r},t)$ and the Poynting vector $\boldsymbol{S}(\boldsymbol{r},t)$ for the evanescent field below the prism, then confirm the energy continuity equation $\boldsymbol{\nabla}\cdot\boldsymbol{S}(\boldsymbol{r},t) + \partial\mathcal{E}(\boldsymbol{r},t)/\partial t = 0$.

2 Pts c) Show that the component of the evanescent field's time-averaged Poynting vector along the z-axis is zero, while that along the x-axis is non-zero.

2 Pts d) Find the time-averaged areal energy density (per unit area of the xy-plane) stored in the evanescent field.

Problem 3) The figure shows an infinitely large, thin, planar sheet having a uniform oscillating magnetization $\boldsymbol{M}(\boldsymbol{r},t) = M_{so}\delta(y)\cos(\omega_o t)\hat{\boldsymbol{z}}$.



2 Pts a) What are the units of M_{so}?

2 Pts b) Find the bound electric charge- and current-densities $(\rho_{\text{bound}}^{(e)}, \boldsymbol{J}_{\text{bound}}^{(e)})$ associated with the magnetic sheet.

2 Pts c) Using the standard formula

$$\boldsymbol{A}(\boldsymbol{r},t) = (\mu_o/4\pi)\int_{-\infty}^{\infty} [\boldsymbol{J}(\boldsymbol{r}', t - |\boldsymbol{r}-\boldsymbol{r}'|/c)/|\boldsymbol{r}-\boldsymbol{r}'|]\,\mathrm{d}\boldsymbol{r}',$$

find the distribution of the vector potential throughout the entire space-time.

2 Pts d) Determine the field distributions $\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{H}(\boldsymbol{r},t)$ everywhere in space and time.

Hint: $\int_0^\infty \frac{\sin\left(p\sqrt{x^2+a^2}\right)}{\sqrt{x^2+a^2}}\mathrm{d}x = (\pi/2)J_0(pa);$ $\quad a>0,\ p>0.$ (G&R 3.876 – 1)

$\int_0^\infty \frac{\cos\left(p\sqrt{x^2+a^2}\right)}{\sqrt{x^2+a^2}}\mathrm{d}x = -(\pi/2)Y_0(pa);$ $\quad a>0,\ p>0.$ (G&R 3.876 – 2)

$\int_0^\infty J_0\left(p\sqrt{x^2+a^2}\right)\mathrm{d}x = p^{-1}\cos(pa);$ $\quad a>0,\ p>0.$ (G&R 6.677 – 3)

$\int_0^\infty Y_0\left(p\sqrt{x^2+a^2}\right)\mathrm{d}x = p^{-1}\sin(pa);$ $\quad a>0,\ p>0.$ (G&R 6.677 – 4)

Problem 4) A static electric point-quadrupole located at the origin of a Cartesian coordinate system has the following charge-density distribution: $\rho(\boldsymbol{r},t) = Q\delta'(x)\delta'(y)\delta(z)$.

2 Pts a) Draw a simple schematic diagram to indicate how the positive and negative charges of the quadrupole are distributed in the vicinity of the origin.

(**Hint**: Whereas a dipole has one positive and one negative charge, a quadrupole consists of two positive and two negative charges.)

2 Pts b) What are the units of Q, the parameter that determines the strength of the quadrupole?

2 Pts c) Using the formula $\psi(\boldsymbol{r}) = (4\pi\varepsilon_o)^{-1}\int_{-\infty}^{\infty}[\rho(\boldsymbol{r}')/|\boldsymbol{r}-\boldsymbol{r}'|]\mathrm{d}\boldsymbol{r}'$, calculate the scalar potential of the point-quadrupole in its surrounding space.

2 Pts d) Express $\psi(\boldsymbol{r})$ in spherical coordinates, then calculate the electric field distribution $\boldsymbol{E}(\boldsymbol{r})$ produced by the point-quadrupole.
