Day 1) Denoting by $\boldsymbol{\sigma}$ the normalized k-vector $\boldsymbol{k}/k_0 = c\boldsymbol{k}/\omega_0 = \lambda_0\boldsymbol{k}/2\pi$, we write

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_0 \exp[\mathrm{i}(2\pi/\lambda_0)(\boldsymbol{\sigma}\cdot\boldsymbol{r} - ct)], \quad \boldsymbol{H}(\boldsymbol{r},t) = \boldsymbol{H}_0 \exp[\mathrm{i}(2\pi/\lambda_0)(\boldsymbol{\sigma}\cdot\boldsymbol{r} - ct)].$$

Maxwell's first and fourth equations then yield

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = 0 \quad\rightarrow\quad \boldsymbol{\sigma}\cdot\boldsymbol{E}_0 = 0 \qquad \text{and} \qquad \boldsymbol{\nabla}\cdot\boldsymbol{B} = \mu_0\boldsymbol{\nabla}\cdot\boldsymbol{H} = 0 \quad\rightarrow\quad \boldsymbol{\sigma}\cdot\boldsymbol{H}_0 = 0.$$

Invoking Maxwell's second and third equations, we now find

$$\boldsymbol{\nabla}\times\boldsymbol{E} = -\,\partial\boldsymbol{B}/\partial t \quad\rightarrow\quad \mathrm{i}(2\pi/\lambda_0)\boldsymbol{\sigma}\times\boldsymbol{E}_0 = -(-\mathrm{i}2\pi c/\lambda_0)\mu_0\boldsymbol{H}_0 \;\rightarrow\; \boldsymbol{\sigma}\times\boldsymbol{E}_0 = Z_0\boldsymbol{H}_0.$$

$$\boldsymbol{\nabla}\times\boldsymbol{H} = \partial\boldsymbol{D}/\partial t \quad\rightarrow\quad \mathrm{i}(2\pi/\lambda_0)\boldsymbol{\sigma}\times\boldsymbol{H}_0 = (-\mathrm{i}2\pi c/\lambda_0)\varepsilon_0\varepsilon\boldsymbol{E}_0 \;\rightarrow\; \boldsymbol{\sigma}\times\boldsymbol{H}_0 = -(\varepsilon/Z_0)\boldsymbol{E}_0.$$

Combining the above equations, we will have

$$\boldsymbol{\sigma}\times(\boldsymbol{\sigma}\times\boldsymbol{E}_0) = -\varepsilon\boldsymbol{E}_0 \quad\rightarrow\quad \overbrace{(\boldsymbol{\sigma}\cdot\boldsymbol{E}_0)}^{0}\boldsymbol{\sigma} - (\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})\boldsymbol{E}_0 = -\varepsilon\boldsymbol{E}_0 \quad\rightarrow\quad \boldsymbol{\sigma}\cdot\boldsymbol{\sigma} = \varepsilon = n^2.$$

a) In free space: $n = 1 \quad\rightarrow\quad \boldsymbol{\sigma} = \hat{\boldsymbol{z}} \quad\rightarrow\quad Z_0\boldsymbol{H}_\mathrm{i} = \hat{\boldsymbol{z}}\times E_\mathrm{i}\hat{\boldsymbol{x}} \quad\rightarrow\quad Z_0\boldsymbol{H}_\mathrm{i} = E_\mathrm{i}\hat{\boldsymbol{y}}.$

b) Inside the slab: $\boldsymbol{\sigma} = n\hat{\boldsymbol{z}} \quad\rightarrow\quad Z_0\boldsymbol{H}_\mathrm{t} = \boldsymbol{\sigma}\times\boldsymbol{E}_\mathrm{t} = n\hat{\boldsymbol{z}}\times\boldsymbol{E}_\mathrm{t} \quad\rightarrow\quad Z_0\boldsymbol{H}_\mathrm{t} = nE_\mathrm{t}\hat{\boldsymbol{y}}.$

c) Rate of energy flow in free space: $\langle\boldsymbol{S}_\mathrm{i}\rangle = ½\boldsymbol{E}_\mathrm{i}\times\boldsymbol{H}_\mathrm{i} = (E_\mathrm{i}^2/2Z_0)\hat{\boldsymbol{z}}.$

Rate of energy flow in the transparent slab: $\langle\boldsymbol{S}_\mathrm{t}\rangle = ½\boldsymbol{E}_\mathrm{t}\times\boldsymbol{H}_\mathrm{t} = (nE_\mathrm{t}^2/2Z_0)\hat{\boldsymbol{z}}.$

$$\langle\boldsymbol{S}_\mathrm{i}\rangle = \langle\boldsymbol{S}_\mathrm{t}\rangle \quad\rightarrow\quad E_\mathrm{i}^2/2Z_0 = nE_\mathrm{t}^2/2Z_0 \quad\rightarrow\quad E_\mathrm{t} = E_\mathrm{i}/\sqrt{n} \quad\rightarrow\quad H_\mathrm{t} = nE_\mathrm{t}/Z_0 = \sqrt{n}E_\mathrm{i}/Z_0.$$

d) E-field energy-density inside dispersionless medium: $¼\varepsilon_0\varepsilon E_\mathrm{t}^2 = ¼\varepsilon_0 n^2(E_\mathrm{i}/\sqrt{n})^2 = ¼\varepsilon_0 nE_\mathrm{i}^2.$

H-field energy-density inside dispersionless medium: $¼\mu_0 H_\mathrm{t}^2 = ¼\mu_0(\sqrt{n}\,E_\mathrm{i}/Z_0)^2 = ¼\varepsilon_0 nE_\mathrm{i}^2.$

Thus, the E- and H-field energy-densities within the transparent medium of the slab are equal.

Let the pulse duration and cross-sectional area be T and A, respectively. In the free-space region, the length of the pulse is cT, its volume is cTA, and its energy-density is $¼\varepsilon_0E_\mathrm{i}^2 + ¼\mu_0H_\mathrm{i}^2 = ¼\varepsilon_0E_\mathrm{i}^2 + ¼\mu_0(E_\mathrm{i}/Z_0)^2 = ½\varepsilon_0E_\mathrm{i}^2$. Consequently, the total energy of the light pulse in the free-space region is $½\varepsilon_0E_\mathrm{i}^2cTA$.

Inside the glass medium of the dielectric slab, the length of the pulse is cT/n, its volume is cTA/n, and its energy-density is $¼\varepsilon_0nE_\mathrm{i}^2 + ¼\varepsilon_0nE_\mathrm{i}^2 = ½\varepsilon_0nE_\mathrm{i}^2$. Therefore, the total energy of the pulse propagating within the slab is also $½\varepsilon_0E_\mathrm{i}^2cTA$. The pulse energy is thus seen to be preserved.

Day 2) a) $\boldsymbol{H}(x,t) = H_0 \cos\{\omega[t - n(\omega)x/c]\}\,\hat{\boldsymbol{z}}$.

Maxwell's third equation: $\boldsymbol{\nabla} \times \boldsymbol{E} = -\,\partial \boldsymbol{B}/\partial t \quad \rightarrow \quad (\partial E_y/\partial x)\hat{\boldsymbol{z}} = -\mu_0(\partial H_z/\partial t)\hat{\boldsymbol{z}}$

$$\rightarrow \quad E_0[\omega n(\omega)/c]\sin\{\omega[t - n(\omega)x/c]\} = \mu_0 H_0 \omega \sin\{\omega[t - n(\omega)x/c]\}$$

$$\rightarrow \quad H_0 = n(\omega)E_0/\mu_0 c = n(\omega)E_0/Z_0.$$

b)
$$\boldsymbol{S}(x,t) = \boldsymbol{E}(x,t) \times \boldsymbol{H}(x,t) = E_0 H_0 \cos^2\{\omega[t - n(\omega)x/c]\}\,\hat{\boldsymbol{x}}$$

$$\rightarrow \quad \langle \boldsymbol{S}(x,t)\rangle = E_0 H_0 \langle \cos^2\{\omega[t - n(\omega)x/c]\}\rangle \hat{\boldsymbol{x}} = [n(\omega)/2Z_0]E_0^2\hat{\boldsymbol{x}}.$$

c)
$$\boldsymbol{E}_1(x,t) + \boldsymbol{E}_2(x,t) = E_0\{\cos\{\omega[t - n(\omega)x/c]\} + \cos\{\omega'[t - n(\omega')x/c]\}\}\hat{\boldsymbol{y}}$$

$$= 2E_0 \cos\left\{\left(\frac{\omega + \omega'}{2}\right)t - \frac{[\omega n(\omega) + \omega' n(\omega')]x}{2c}\right\}\cos\left\{\left(\frac{\omega' - \omega}{2}\right)t - \frac{[\omega' n(\omega') - \omega n(\omega)]x}{2c}\right\}\hat{\boldsymbol{y}}$$

$$\cong 2E_0 \underbrace{\cos\{\omega_c[t - n(\omega_c)x/c]\}}_{\substack{\text{carrier:}\\ \text{phase velocity} = c/n(\omega_c)}} \underbrace{\cos\left\{½\Delta\omega\left[t - \frac{\omega' n(\omega') - \omega n(\omega)}{\omega' - \omega}(x/c)\right]\right\}}_{\substack{\text{envelope:}\\ \text{group velocity} = \frac{c}{\mathrm{d}[\omega n(\omega)]/\mathrm{d}\omega|_{\omega=\omega_c}}}}\hat{\boldsymbol{y}}.$$

Here, $\mathrm{d}[\omega n(\omega)]/\mathrm{d}\omega|_{\omega=\omega_c} = n(\omega_c) + \omega_c n'(\omega_c)$, where the derivative n' of the refractive index n is evaluated at the center frequency ω_c.